C 20208 **(Pages : 3)** **Name..................................**

Reg. No...............................

SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

(2014 to 2018 Admissions)

Time : Three Hours   Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark*.

1. A complex function $f(z)$ is analytic at a point $z = z_0$ if ————.
2. An analytic function with constant argument is ————.
3. Give an example of a complex function which is Differentiable at a point but not analytic at that point.
4. Find the simple poles, if any for the function $f(z) = \dfrac{(z+2)^2}{z^5\left(x^4-1\right)}$.
5. Write the polar form of Cauchy-Riemann equations.
6. Define residue of a complex valued function.
7. Fill in the blanks : The real part of sinh $(2z)$ is ————.
8. Fill in the blanks : $f(z) = e^z$ is periodic with period = ————.
9. A point $z = z_0$ is a singular point of a complex function $w = f(z)$ if ————.
10. Fill in the blanks : $\text{Res}_{z=\pi/2} \tan z =$ ————.
11. The solution of the equation $e^z = -3$ is ————.
12. The principal value of i^i is ————.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Show that $f(z) = \sin z$ is analytic for all z.
14. Find the principal value of $(1-i)^{1+i}$.
15. Show that $\tanh^{-1}(z) = \dfrac{1}{2}\log\dfrac{1+z}{1-z}$.
16. Show that the zeros of an analytic function are isolated.

Turn over

17. Determine and classify the singular points of $f(z)=\frac{(z+2)^2}{z^5\left(z^4-1\right)}$.

18. Find the radius of convergence of the power series : $\sum_{n=0}^{\infty}\frac{n!z^n}{n^n}$.

19. Verify Cauchy-Groursat theorem for $f(z) = z^5$ when the contour of integration is the circle with centre at origin and radius 3 units.

20. Discuss the nature of singularities if any, of $f(z) = \sin(1/z)$ in the complex plane.

21. Find all the solution of $e^z = 2$.

22. Find the residue of $f(z) = \cot(z)$ at its poles.

23. Evaluate $\oint_C \frac{\sin\pi z}{\left(z^6\right)}dz$ around $C = |z| = 1$.

24. Find the Taylor series expansion of $f(z) = e^z$ around $z = i\pi/2$.

25. Evaluate $\oint_{|z|=2}\bar{z}dz$.

26. Illustrate entire function by an example.

(10 × 4 = 40 marks)

Section C

Answer any **six** *questions.*
Each question carries 7 marks.

27. Evaluate $\oint_C \frac{1}{(z-1)(z-2)}$ around the simple closed curve $C = |z| = 4$.

28. Determine the nature of the singularities of the function $f(z) = \sec(1/z)$.

29. Expand $f(z)=\frac{1}{(z+1)(z+2)}$ as a Laurent series valid for $0<|z+1|<2$.

30. If $f(z)=u(x,y+iv(x,y))$ is analytic in a domain D, then prove that its component functions are harmonic in D.

31. Find the analytic function $f(z)$ is terms of z, if $u(x,y)=\text{Re}(f(z))=e^x(x\cos y-y\sin y)$.

32. Show that the function $f(z)=\sqrt{xy}$ is not analytic at the origin, even though Cauchy Riemann equations are satisfied at that point.

33. State and prove Morera's theorem.

34. Show that the derived series has the same radius of convergence as the original series.

35. Evaluate $\oint_{|z-2|=2}\frac{z^3}{(z-1)^4(z-2)(z-3)}dz$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *questions.*
Each question carries 13 marks.

36. (a) State and prove Cauchy's integral formula.

    (b) Prove or disprove : $|\cos(z)| \leq 1$ for all complex numbers z. Justify your claim.

37. (a) State and prove fundamental theorem of Algebra.

    (b) Find the residues of $f(z) = \dfrac{z^2}{(z-1)^2(z-2)}$ at its poles.

38. (a) Evaluate using the method of residues : $\int_0^{2\pi} \dfrac{1}{a+\cos\theta} d\theta$.

    (b) Evaluate $\int_0^{\infty} \dfrac{1}{x^4 + a^4} dx, a > 0$.

(2 × 13 = 26 marks)

C 20208-A **(Pages : 6)** **Name....................................**

Reg. No.................................

SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2022

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

(2014 to 2018 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 30**	**Maximum : 30 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 30.
2. The candidate should check that the question paper supplied to him/her contains all the 30 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 6B 10—COMPLEX ANALYSIS

(Multiple Choice Questions for SDE Candidates)

1. Real part of $f(z) = \log z$ is :

   (A) $\frac{1}{2}\log\left(x^2+y^2\right)$. (B) $\log\left(x^2+y^2\right)$.

   (C) $\log(x+iy)$. (D) None of these.

2. If n is a positive integer, then $\left(1+i\sqrt{3}\right)^n+\left(1-i\sqrt{3}\right)^n$ is equal to :

   (A) $2^n\sin\frac{n\pi}{2}$. (B) $2^{n+1}\cos\frac{n\pi}{3}$.

   (C) $2^{n+1}\sin\frac{n\pi}{3}$. (D) None of these.

3. Real part of $f(z) = z^3$ is :

   (A) $x^3 - 3xy^2$. (B) $x^3 + 3xy^2$.

   (C) $x^3 - 3x^2y$. (D) None of these.

4. Value of $(1+i)^{24}$ is :

   (A) 2^{24}. (B) $\left(\sqrt{2}\right)^{24} e^{\frac{i\pi}{4}}$.

   (C) 2^{12}. (D) None of these.

5. If $f(z)$ is a real valued analytic function in a domain D, then :

   (A) $f(z)$ is a constant. (B) $f(z)$ is identically zero.

   (C) $f(z)$ has modulus 1. (D) None of these.

6. The function e^{iz} has period :

   (A) $2\pi i$. (B) 2π.

   (C) π. (D) 3π.

7. For real numbers x and y, $\sin(x+iy)$ equals :

   (A) $\sin x\cosh y + i\cos x\sinh y$. (B) $\cos x\cosh y - i\sin x\sinh y$.

   (C) $\sin x\cosh y - i\cos x\sinh y$. (D) $\cos x\cosh y + i\sin x\sinh y$.

8. Real part of the function $f(z) = |z|^2$ equals :

(A) $2xy$.
(B) $x^2 - y^2$.
(C) $x^2 + y^2$.
(D) None of these.

9. Which of the following is not a simply connected region ?

(A) Circular disk.
(B) Half planes.
(C) An annulus region.
(D) A parallel strip.

10. The integral $\int_{|z|=2\pi} \frac{\sin z}{(z-\pi)^2} dz$ where the curve is taken anti-clockwise, equals :

(A) $-2\pi i$.
(B) $2\pi i$.
(C) 0.
(D) $4\pi i$.

11. The value of the integral $\int_C \frac{dz}{(z-a)^{10}}$, where C is $|z-a| = 3$ is :

(A) 0.
(B) $2\pi i$.
(C) πi.
(D) None of these.

12. The value of the integral $\int_C \frac{e^{5z}}{z^3} dz$, where C is $|z| = 3$ is :

(A) $10\pi i$.
(B) $2\pi i$.
(C) $25\pi i$.
(D) None of these.

13. Value of the integral $\int_0^x e^{it} dt$ is :

(A) $2i$.
(B) 0.
(C) $2\pi i$.
(D) None of these.

14. If n is any non-zero integer, then $\int_0^{2\pi} e^{in\theta} d\theta$ equals :

(A) 0.
(B) 2π.
(C) 1.
(D) None of these.

15. Converse of Cauchy's integral theorem is known as :

(A) Liouville's theorem.
(B) Goursat's theorem.
(C) Morera's theorem.
(D) Euler's theorem.

16. A Maclaurin series is a Taylor series with centre :

(A) $z_0 = 1$. (B) $z_0 = 0$.

(C) $z_0 = 2$. (D) None of these.

17. The radius of convergence of the power series of the function $f(z) = \frac{1}{1-z}$ about $z = 1/4$ is :

(A) 1. (B) ¼.

(C) ¾. (D) 0.

18. If $f(z)$ is entire, then $f(z) = \sum_{n=1}^{\infty} a_n z^n$ has radius of convergence :

(A) 0. (B) e.

(C) ∞. (D) None of these.

19. A power series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$ always converges for :

(A) at least one point z.

(B) all complex numbers z.

(C) at all z which are either real or purely imaginary.

(D) at all z with $|z - z_0| < R$ for some $R > 0$.

20. A function $f(z)$ given by a power series is analytic at :

(A) Every point of its domain.

(B) Every point inside its circle of convergence.

(C) Every point on the circle of convergence.

(D) Every point in the complex plane.

21. The singular points of the function $f(z) = \frac{1}{4z - z^2}$ are :

(A) $z = 0$ and $z = -4$. (B) $z = 0$ and $z = 4$.

(C) $z = 4$ and $z = -4$. (D) $z = 2$ and $z = -2$.

22. The constant term in the Laurent series expansion of $f(z) = \frac{e^z}{z^2}$ in the region $0 < |z| < \infty$ is :

(A) 0. (B) ½.

(C) 2. (D) None of these.

23. If $f(z)$ has a zero of order m at z_0 and $g(z)$ has a pole of order n at z_0 and $n \leq m$, then the product $f(z)g(z)$ has at z_0 :

(A) An essential singularity. (B) A pole of order $m - n$.

(C) A removable singularity. (D) A pole of order $m - 1$.

24. If $f(z)$ has a pole of order m at z_0, then $g(z) = \frac{f'(z)}{f(z)}$, at z_0 has :

(A) A simple pole. (B) A pole of order m.

(C) A pole of order $m + 1$. (D) A pole of order $m - 1$.

25. For $f(z) = \frac{\tan z}{z}, z = 0$ is a :

(A) Essential singularity. (B) Simple pole.

(C) Removable singularity. (D) Double pole.

26. Which of the following function has a simple zero at $z = 0$ and an essential singularity $z = 1$?

(A) $ze^{\frac{1}{z-1}}$. (B) $ze^{\frac{1}{1+z}}$.

(C) $(z-1)e^{\frac{1}{z}}$. (D) $(z-1)e^{\frac{1}{z-1}}$.

27. The function $f(z) = \frac{z^2 + 2iz + 3}{(z-i)^2(z+i)}$ at $z = i$ has :

(A) Regular point. (B) Simple pole.

(C) Double pole. (D) Removable singularity.

28. Singularities of a rational function are :

(A) Poles. (B) Essential.

(C) Non-isolated. (D) Removable.

29. The singularity of the function $\frac{\sin z}{z}$ at $z = 0$ is :

(A) Essential singularity.

(B) Simple pole.

(C) Removable singularity.

(D) Double pole.

30. At $z = 1$, the residue of $f(z) = \frac{z^2 + 1}{z(z-1)}$ is :

(A) – 1.

(B) 0.

(C) 1.

(D) 2.