D 10233 **(Pages : 3)** **Name..................................**

Reg. No...............................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS–UG)

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

Time : Three Hours Maximum : 120 Marks

Part A

Answer **all** *questions.*
Each question carries 1 *mark*.

1. Prove that the product of two odd functions is an even function.
2. Prove $L^{-1}\{1\}$.
3. Write down the differential equation whose solution is $y = c_1 e^{5t} + c_2 e^{-2t}$.
4. Evaluate $W\left[e^{\mu \cos \lambda t}, e^{\mu \sin \lambda t}\right]$.
5. Compute $L\left\{t^2 e^{\lambda t}\right\}$.
6. Find the integrating factor of $(x-2)(x+1)\dfrac{dy}{dx} + 3y = x$.
7. Solve the system $\dfrac{dy}{dt} - x = 0, \dfrac{dx}{dt} - y = 0$.
8. Find the fundamental solutions of $y'' + 25y = t^{-1/2}$.
9. Find the value of b_n in the Fourier sine series expansion of 2π-periodic function $f(x) = -x, x \in [-\pi, \pi]$.
10. Write one dimensional heat equation with all the assumptions involved.
11. What do you mean by an exact differential equation ? Give an example.
12. Find the complementary function corresponding to $y'' - 2y' + 2y = t$.

(12 × 1 = 12 marks)

Part B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Convert $y'' + 2y' = 0$ into a system of first order equations.

Turn over

14. Find the Fourier cosine series for the 2π-periodic function $f(x) = -x, x \in [-\pi, \pi]$.
15. Find the integrating factor for $(2x+3y)\,dx + (2x-3y)\,dy = 0$.
16. Find the inverse Laplace transform of $\log\left((s-a)/(s-b)\right)$.
17. Define unit step function and find its Laplace transform.
18. Solve : $t^2x'' - 2tx' - 3x = 0$.
19. Write the existence and uniqueness theorem for first order differential equations with the assumptions involved therein.
20. Show that the inverse Laplace transform is linear.
21. State Abel's theorem.
22. Solve : $\dfrac{dy}{dx} = (3x+2y+1)^2$.
23. Evaluate $L\left\{te^t \cos 2t\right\}$.
24. Find the second order p.d.e. for which $y = \phi(x+at) + \psi(x-at)$ is a solution.
25. Solve the system : $\dfrac{dy}{dt} = x - y, \dfrac{dx}{dt} = x + y$.
26. Solve : $y' - 2y = 0$ using Laplace transform.

(10 × 4 = 40 marks)

Part C

Answer any **six** *questions.*
Each question carries 7 marks.

27. Express the function $f(t) = \begin{cases} t\sin t, & \text{if } 0 \le t < \pi/2 \\ \cos t, & \text{if } \pi/2 \le t < \pi \\ 0, & \text{elsewhere} \end{cases}$ in terms of combination of unit step functions and hence find its Laplace transform.
28. Evaluate the Laplace inverse transforms of $4 - \cot^{-1}(s/a)$ and $\dfrac{1}{\left(s^2-5s+6\right)^2}$.
29. Find the solution by the checking the exactness of $\left(3y^2 - 2xy + 2\right)dx + \left(6xy - x^2 + y^2\right)dy = 0$.
30. State the conditions for the existence of Laplace transform of a function $f(t)$ and prove the same.

31. A ball with mass 0.15 kg. is thrown upward with initial velocity 20 m/s from the roof of a building 30 m. high. Neglect air resistance (a) Find the maximum height above the ground that the ball reaches ; (b) Assuming that the ball misses the building on the way down, find the time that it hits the ground.

32. Find the Fourier cosine series for the function $f(t) = \pi|t-\pi|, t \in [0, \pi]$.

33. Find the solution of the heat conduction problem :

$$25u_{xx} = u_t, 0 < x < 1, t > 0; u(0,t) = 0, u(1,t) = 0, t > 0; u(x,0) = \sin(2\pi x) - \sin(5\pi x), 0 \le x \le 1.$$

34. State and prove Convolution theorem for Laplace transforms.

35. Evaluate (i) $L^{-1}\left(\frac{1-e^{-s}}{s}\right)$ and (ii) get a formula for $L(f(t))$ where $f(t)$ is a periodic function of period T.

(6 × 7 = 42 marks)

Part D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. (a) Use method of separation of variables and solve the one-dimensional heat equation completely. State the assumptions involved therein explicity.

(b) Find the solution of the p.d.e. $\frac{\partial^2 u}{\partial y\, \partial x} = 2x$.

37. (a) Solve the following differential equation in two ways, one of them must be using Laplace transform. $4y'' - y = t, y(0) = 1, y(1) = 0$.

(b) Find the Fourier series of $f(x) = x^2, x \in [-2, 2]$ treating it as a periodic function of period 4.

38. (a) Apply method of variation of parameters to solve : $y'' - y = \sec t$.

(b) Solve $(2x + y + 3)\, dx + (x - 3y + 2)\, dy = 0$.

(2 × 13 = 26 marks)

D 10233-A **(Pages : 4)** **Name.......................................**

Reg. No...................................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS—UG)

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 30 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 5B 08—DIFFERENTIAL EQUATIONS

(Multiple Choice Questions for SDE Candidates)

1. The degree of the differential equator $(y')^{3/2} = y''$ is ?

(A) 0. (B) 1.

(C) 2. (D) None of these.

2. The integral curves of the differential equation $y' = 1$ are ?

(A) $y = x + c$. (B) $y = x^2 + c$.

(C) $y = x^3 + c$. (D) $y = x + 1$.

3. An integrating factor of the differential equation $ty' + 2y = 4t^2$ is ?

(A) t^3. (B) t^4.

(C) t^2. (D) None of these.

4. A homogeneous differential equation $\frac{dy}{dx} = f\left(y/x\right)$ can be converted to a variable separable equator using a transformation :

(A) $y = vx$. (B) $y^2 = vx$.

(C) $y = vx^2$. (D) $y = v^2x$.

5. The differential equation $\left(6xy^2 + 4x^3y\right) dx + \left(6x^2y + x^4 + e^y\right) dy = 0$ is ?

(A) A separable equation. (B) A linear equation.

(C) An exact equation. (D) A homogeneous linear equation.

6. An integrating factor of the differential equation $3\left(x^2 + y^2\right) dx + x\left(x^2 + 3y^2 + 6ydy = 0\right)$ is :

(A) e^{-y}. (B) e^2y.

(C) e^y. (D) e^{-2y}.

7. The solution of the differential equation $y' = y^2, y(0) = 1$ exists in the region :

(A) $(0, \infty)$. (B) $(-\infty, 0)$.

(C) $(-\infty, 1)$. (D) $(-\infty, \infty)$.

8. If $\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)$ is a function of y only, then an integrating factor of the differential equation Mdx + Ndy = 0 is :

(A) $\mu(x)=\exp\left[\int\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dy\right]$. (B) $\mu(x)=\exp\left[\int\frac{1}{M}\left(\frac{\partial N}{\partial x}+\frac{\partial M}{\partial y}\right)dy\right]$.

(C) $\mu(x)=\int\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dy$. (D) $\mu(x)=\int\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dy$.

9. An integrating factor of the differential equation $\frac{dx}{dy}+P(x)=Q(x)$ is :

(A) $e^{\int pdx}$. (B) $e^{-\int pdx}$.

(C) $e^{\int p^2dx}$. (D) $e^{\int (P+Q)dx}$.

10. An integrating factor of the differential equator $\frac{dx}{dy}+Px=Q$ where P and Q are functions of y alone is :

(A) $e^{\int pdy}$. (B) $e^{-\int pdy}$.

(C) $e^{\int pdx}$. (D) $e^{\int p^2dy}$.

11. The initial value problem $y'=y^{y_3}, y(0)=0, t\geq 0$:

(A) A unique solution. (B) Infinitely many solutions.

(C) No solution. (D) Two solutions.

12. The general solution of the differential equation $3\left(x^2+y^2\right)dx+x\left(x^2+3y^2+6ydy=0\right)$ is :

(A) $x^3 e^{-y}+3xy^2 e^y=c$. (B) $x^3 e^y+3xy^2 e^y=c$.

(C) $x^2 e^y+3x^2y^2 e^y=c$. (D) $xe^y+ye^y=c$.

13. If $y_1(x)$ and $y_2(x)$ are two linearly independent solutions of the linear differential equation $a_0(x)y''+a_1(x)y'+a_2+(x)y=0$ then :

(A) $y_1(x)y_2(x)$ is also a solution. (B) $y_1(x)+y_2(x)$ is also a solution.

(C) $y_1(x)/y_2(x)$ is also a solution. (D) $y_1^2(x)+y_2^2(x)$ is also a solution.

14. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solutions of the differential equation $a_0(x)\,y'' + a_1(x)\,y' + a_2(x)\,y = 0$ then the Wronskian $w(y_1, y_2)$ is :

(A) 1. (B) 0.

(C) 2. (D) –1.

15. The general solution of the differential equation $\left(D^2 - 4D + 4\right)y = 0$ is :

(A) $(c_0 + c_1 x)e^{2x}$. (B) $(c_0 - c_1 x)e^{2x}$.

(C) $c_1 e^x\ c_2 e^{-2x}$. (D) None of these.

16. The Laplace transform of e^{at} is :

(A) $\frac{1}{s-a}$. (B) $\frac{1}{s+a}$.

(C) $\frac{1}{s^2-a^2}$. (D) $\frac{1}{s^2-a^2}$.

17. The Laplace transform of cos at is :

(A) $\frac{s}{s^2+a^2}$. (B) $\frac{1}{s^2+a^2}$.

(C) $\frac{1}{s^2-a^2}$. (D) $\frac{s}{s^2-a^2}$.

18. If $L\{f(t)\} = F(s)$, then $L\{f(at)\} =$

(A) $\frac{1}{a} f(s/a)$. (B) $F(s/a)$.

(C) $F(a/s)$. (D) $F(s)$.

19. The Laplace transform of the delta function is :

(A) e^{-as}. (B) e^{as}.

(C) $e^{as/s}$. (D) $e^{-as/s}$.

20. $\int_0^\infty \frac{\sin t}{t}\, dt =$

(A) $\frac{\pi}{4}$. (B) $\frac{\pi}{8}$.

(C) $\frac{\pi}{2}$. (D) None of these.