

C 41232

(Pages : 3)

Name.....

Reg. No.....

**FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2023**

Mathematics

MTS 4C 04—MATHEMATICS—4

(2019 Admission onwards)

Time : Two Hours

Maximum : 60 Marks

Section A

Answer any number of questions.

Each question carries 2 marks.

Ceiling is 20.

1. Solve the initial value problem $\frac{dy}{dx} = \frac{-x}{y}$, $y(4) = -3$.
2. Solve $(x^2 - 9) \frac{dy}{dx} + xy = 0$.
3. Find the value of k so that the differential equation $(y^3 + kxy^4 - 2x) dx + (3xy^2 + 20x^2y^3) dy = 0$ is exact.
4. Verify that the functions e^{-3x} , e^{4x} form a fundamental set of solutions of the differential equation $y'' - y' - 12y = 0$ on $(-\infty, \infty)$.
5. The function $y_1 - e^x$ is a solution of $y'' - y = 0$ on the interval $(-\infty, \infty)$, use reduction of order to find a second solution y_2 .
6. Find the general solution of $y''' - 4y'' - 5y' = 0$.
7. Solve the initial value problem $y'' + 4y' + 5y = 35e^{-4x}$, $y(0) = -3$, $y'(0) = 1$.
8. Find $\mathcal{L}(f(t))$, where $f(t) = \sin 2t \cos 2t$.
9. Evaluate $\mathcal{L}^{-1} \left(\frac{s}{(s-2)(s-3)(s-6)} \right)$.

Turn over

10. Write $f(t) = \begin{cases} 2, & 0 \leq t < 3 \\ -2, & t \geq 3 \end{cases}$ in terms of unit step functions and find $\mathcal{L}(f(t))$
11. Show that the functions $f_1(x) = e^x$, $f_2(x) = xe^{-x} - e^{-x}$ are orthogonal on $[0, 2]$,
12. Show that the partial differential equation $\frac{\partial^2 u}{\partial x^2} + 6 \frac{\partial^2 u}{\partial x \partial y} + 9 \frac{\partial^2 u}{\partial y^2}$ is parabolic.

Section B

*Answer any number of questions.
Each question carries 5 marks. Ceiling is 30.*

13. Solve $x \frac{dy}{dx} + y = x^2 y^2$.
14. Solve $\frac{dy}{dx} = (x + y + 1)^2$ by using an appropriate substitution.
15. $y''' + y'' = e^x \cos x$.
16. Solve $x^2 y'' - 3xy' + 3y = 2x^4 e^x$.
17. Solve $y' + 6y = e^{4t}$, $y(0) = 2$ using the Laplace transform.
18. Evaluate $\mathcal{L}^{-1} \left(\frac{1}{(s^2 + k^2)^2} \right)$.
19. Expand $f(x) = x$, $-2 < x < 2$ in a Fourier series.

Section C

*Answer any **one** question.*

The question carries 10 marks.

20. Solve the initial value problem $y'' - 6y' + 9y = t^2 e^{3t}$, $y(0) = 2$, $y'(0) = 17$ using Laplace transform.

21. Expand $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi - x, & 0 \leq x < \pi \end{cases}$ in a Fourier series.