D 30572 **(Pages : 2)** **Name.....**

Reg. No.....

FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2022

Mathematics

MTS 5B 09—INTRODUCTION TO GEOMETRY AND THEORY OF EQUATIONS

(2020 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 *marks. Ceiling is* 20.

1. Does the equation $3x^2 - 10xy + 3y^2 + 14x - 2y + 3 = 0$ represents a hyperbola ? Justify your answer.

2. Find the vertex and directrix of the parabola $y^2 = 4x$.

3. Show that the parametric equation $x = 3\cos t, y = 2\sin t; -\pi < t \leq \pi$ represents the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

4. State the reflection property of the parabola.

5. Find the reminder when $f(x) = x^7 - 7x^3 + 1$ is divisible by $x + 1$.

6. Find the sum of the squares of the roots of the equation $x^4 - 2x + 1 = 0$.

7. State the Fundamental Theorem of Algebra.

8. Solve the equation $(a-b)x^2 - (b-c)x + (c-a) = 0$.

9. Find Δ of the equation $x^3 + 10x - 7 = 0$.

10. Show that $\sqrt[3]{\sqrt{5}+2} - \sqrt[3]{\sqrt{5}-2} = 1$.

Turn over

11. Show that the equation $x^6 - x^4 + 4x - 11 = 0$ has a root lies between 1 and 2.

12. Find the real root of the cubic equation $x^3 + 9x - 2 = 0$.

Section B

Answer any number of questions.
Each question carries 5 *marks. Ceiling is* 30.

13. (i) State the Fundamental theorem of Affine Geometry.

    (ii) Determine the affine transformation which maps the points $(0, 0), (1, 0)$ and $(0, 1)$ to the points $(3, 2), (5, 8)$ and $(7, 3)$ respectively.

14. Determine the image of the line $y = -x$ under the affine transformation $t(x) = \begin{pmatrix} 4 & 1 \\ 2 & 1 \end{pmatrix} \times + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

15. Solve $x^3 - 9x^2 + 26x - 24 = 0$ if the roots form an arithmetic progression $\alpha - \beta, \alpha, \alpha + \beta$.

16. Factorize into real linear and quadratic factors of the polynomial $f(x) = x^4 + 9$.

17. Find the rational roots of the equation $4x^3 - 4x^2 - x + 1 = 0$.

18. Separate the roots of the equation $2x^5 - 5x^4 + 10x^2 - 10x + 1 = 0$.

19. Show that the necessary and sufficient condition for an equation $x^3 + px + q = 0$ to have three real and distinct roots is $4p^3 + 27q^2 < 0$.

Section C

Answer any **one** *question.*
The question carries 10 *marks.*

20. (i) Prove that a perpendicular from a focus of a parabola to a tangent meets the tangent on the directrix of the parabola.

    (ii) Determine the equation of the tangent to the ellipse with parametric equations $x = 3\cos t, y = \sin t$ at the point with parameter $t = \pi/4$. Deduce the co-ordinates of the point of intersection of this tangent with the x-axis.

21. Solve the cubic equation $x^3 - 3x^2 + 12x + 16 = 0$ by using Carden's formula.

(1 × 10 = 10 marks)