D 13605 **(Pages : 3)** **Name.....**

Reg. No.....

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MTS 1C 01—MATHEMATICS—I

(2019—2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 *marks.*
Maximum 20 *marks.*

1. Find the derivative of $f(x) = x^2 - x$ at $x = 2$.

2. Find $\lim_{x \to 0} \frac{\sqrt{x^2 + 100} - 10}{x^2}$.

3. Find the tangent line to the curve $y = \sqrt{x}$ at $x = 4$.

4. Find the derivative of $y = \left(x^2 + 1\right)\left(x^3 + 3\right)$.

5. Give the parameterization of the circle $x^2 + y^2 = 1$.

6. Find $\lim_{y \to 1} \sec\left(y \sec^2 y - \tan^2 y - 1\right)$.

7. Suppose that $F'(x) = x$ for all x and that $F(3) = 2$. What is $F(x)$?

8. Suppose that f is differentiable on the whole real line and that $f'(x)$ is constant. Prove that f is linear.

9. Prove that for the curve $y = c \sin \frac{x}{a}$, every point at which it meets the x-axis is a point of inflection.

Turn over

10. Find the maximum and minimum points and values for the function $f(x) = \left(x^2 - 8x + 12\right)^4$ on the interval $[-10, 10]$.

11. Find $\sum_{k=1}^{7} \left(3 - k^2\right)$.

12. Find $\int_0^1 \frac{\left(3x^2 + x^4\right)}{\left(1 + x^2\right)^2} dx$.

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Maximum 30 *marks.*

13. If $\sqrt{5 - 2x^2} \le f(x) \le \sqrt{5 - x^2}$ for $-1 \le x \le 1$, find $\lim_{x \to 0} f(x)$.

14. Find the linearization of $f(x) = \sqrt{x+1} + \sin x$ at $x = 0$. How is it related to the individual linearizations for $\sqrt{x+1}$ and $\sin x$?

15. An oil slick has area $y = 30x^3 + 100x$ square meters x minutes after a tanker explosion. Find the average rate of change in area with respect to time during the period from $x = 2$ to $x = 3$ from $x = 2$ to $x = 2.1$. What is the instantaneous rate of change of area with respect to time at $x = 2$?

16. Use implicit differentiation to find dy/dx if $6y^2 + \cos y = x^2$.

17. Prove that the curve $y = \frac{x}{1 + x^2}$ has three points of inflection and they are collinear.

18. Evaluate $\lim_{x \to \infty} \frac{x^n}{e^x}$, where n is natural number.

19. Find the area of the region enclosed by the curves $x + y^2 = 3$ and $4x + y^2 = 0$.

Section C

Answer any **one** *question.*
The question carries 10 *marks.*
Maximum 10 *marks.*

20. (a) State and prove the quotient rule of differentiation for positive integers.

(b) Prove that $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, (n \neq -1)$.

(c) A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder.The second plane crosses the first plane at 45° angle at the center of the cylinder. Find the volume of the wedge.

21. (a) On what interval is $f(x) = x^3 - 2x + 6$ increasing or decreasing ?

(b) Find the asymptotes of the graph of $f(x) = -\frac{8}{x^2 - 4}$.

(c) Find the equation of the line tangent to the parametric curve given by the equations $x = (1 + t^3)^4 + t^2, y = t^5 + t^2 + 2$ at $t = 1$.

(1 × 10 = 10 marks)