C 4161 **(Pages : 4)** **Name.....**

Reg. No.....

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
APRIL 2021

Mathematics

MAT 2B 02—CALCULUS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. What is the minimum value of $f(x) = \cos x$, on $[-\pi/2, \pi/2]$.

2. Evaluate $\lim_{x \to \infty} \left(5 + \frac{1}{x}\right)$.

3. Find the average value of $f(x) = 4 - x^2$ on $[0, 3]$.

4. Evaluate $\int_1^{32} x^{-6/5}\, dx$.

5. Evaluate the sum $\sum_{k=1}^{2} \frac{6k}{k+1}$.

6. Suppose that f is integrable and that $\int_1^2 f(x)\, dx = -4$, $\int_1^5 f(x)\, dx = 6$. Evaluate $\int_2^5 f(x)\, dx$.

7. How do you define and calculate the area of the region between the graphs of two continuous functions ?

8. How do you define and calculate the length of the graph of a smooth function over a closed interval ?

9. How do you define and calculate the area of the surface swept out by revolving the graph of a smooth function $y = f(x), a \le x \le b$, about the x-axis ?

Turn over

10. What is the moment about the origin of a thin road along the x-axis with density function $\delta(x)$?

11. Define the work done by a variable force $F(x)$ directed along the x-axis from $x = a$ to $x = b$.

12. State Hooke's Law for springs.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. State the Max-Min Theorem for Continuous Functions.

14. Verify Mean Value Theorem for the function $f(x) = x^2 + 2x - 1$, in the interval $[0, 1]$.

15. Find the linearization of $f(x) = \sqrt{1 + x}$ at $x = 0$.

16. Evaluate $\int_{-4}^{4} |x| \, dx$.

17. Using substitution evaluate the integral $\int_0^3 \sqrt{y + 1} \, dy$.

18. Find the area of the region enclosed by the line $y = 2$ and curve $y = x^2 - 2$.

19. The region between the curve $y = \sqrt{x}$, $0 \le x \le 4$, and the x-axis is revolved about the x-axis to generate a solid. Find its volume.

20. Set up an integral for the length of the curve $y = x^2$, in the interval $-1 \le x \le 2$.

21. Set up an integral for the area of the surface generated by revolving the curve $y = \tan x$, $0 \le x \le \pi/4$; about x-axis.

22. Show that the center of mass of a straight, thin strip or rod of constant density lies halfway between its two ends.

23. Find the work done by a force of $F(x) = 1/x^2$ N along the x-axis from $x = 1\ m$ to $x = 10\ m$.

24. What is the Center of Mass of a thin plate covering a region in the xy-plane ?

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Given $f'(x) = (x-1)^2 (x+2)^2$.

    (a) What are the critical points of f?

    (b) On what intervals is f increasing or decreasing ?

26. Find the asymptotes of the curve :

$$y = \frac{x+3}{x+2}.$$

27. State and prove Rolle's Theorem.

28. Find two positive numbers whose sum is 20 and whose product is as large as possible.

29. Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x, -1 \le x \le 2$.

30. A pyramid 3 m high has a square base that is 3 m on a side. The cross-section of the pyramid perpendicular to the altitude x m down from the vertex is a square x m ona side. Find the volume of the pyramid.

31. Find the length of the curve $y = \frac{4\sqrt{2}}{3} x^{3/2} - 1, \quad 0 \le x \le 1$.

32. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1, x = 4$ about the line $y = 1$.

33. Find the moment about the x-axis of a wire of constant density that lies along the curve $y = \sqrt{x}$ from $x = 0$ to $x = 2$.

(6 × 5 = 30 marks)

Turn over

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. (a) Sketch the Graph of $y = (x-2)^3 + 1$. Include the co-ordinates of inflection point in the graph.

(5 marks)

(b) Find the intervals on which $g(x) = -x^3 + 12x + 5,\ -3 \le x \le 3$ is increasing and decreasing. Where does the function assume extreme values and what are these values ?

(5 marks)

35. (a) If f is continuous at every point of $[a, b]$ and F is any antiderivative of f on $[a, b]$, then prove that

$$\int_a^b f(x)\,dx = \mathrm{F}(b) - \mathrm{F}(a).$$

(5 marks)

(b) A surveyor, standing 30ft from the base of a building, measures the angle of elevation to the top of the building to be 75°. How accurately must the angle be measured for the percentage error in estimating the height of the building to be less than 4 % ?

(5 marks)

36. (a) Find the area of the surface generated by revolving the curve $y = 2\sqrt{x},\ 1 \le x \le 2,$ about the x-axis.

(5 marks)

(b) Find the center of mass of a thin plate of constant density δ covering the region bounded above by the parabola $y = 4 - x^2$ and below by the x-axis.

(5 marks)

[2 × 10 = 20 marks]

C 4161-A **(Pages : 4)** **Name.....**

Reg. No.....

SECOND SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, APRIL 2021

Mathematics

MAT 2B 02—CALCULUS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 20 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 2B 02—CALCULUS

(Multiple Choice Questions for SDE Candidates)

1. A differentiable function is always ————————.

   (A) Continuous. (B) Not continuous.

   (C) Integrable. (D) Not integrable.

2. If $\int_1^2 f(x)\,dx = 5$, then $\int_1^2 f(u)\,du =$ ————————.

   (A) 20. (B) 15.

   (C) 5. (D) 10.

3. What are the critical points of f when $f'(x) = (x-1)(x-2)$?

   (A) 0, 1 and 2. (B) 1 and 2.

   (C) –1 and –2. (D) None of these.

4. Suppose that $\int_2^4 f(x)\,dx = 10$. Find $\int_2^4 -f(x)\,dx$:

   (A) 10. (B) –10.

   (C) 20. (D) –20.

5. One Newton–Metre work is called ————————.

   (A) Newton–Metre. (B) Joule.

   (C) Erg. (D) None of these.

6. $\int_3^3 f(x)\,dx =$ ————————.

   (A) 3. (B) $f(3)$.

   (C) 0. (D) $f(0)$.

7. Find the average value of $f(x) = 2 - x^2$ on [0, 2] :

   (A) 2. (B) –2.

   (C) 4. (D) None of these.

8. $\frac{d}{dx}(\cos x) =$ ————————.

(A) $\sin x$. (B) $-\sin x$.

(C) $-\cos x$. (D) $-\operatorname{cosec} x \cdot \cot x$.

9. $\frac{d}{dy}\left(x^2 + x + 1\right) =$ ————————.

(A) $2x + 1$. (B) $2y + 1$.

(C) 0. (D) 1.

10. $\sum_{k=2}^{4} \frac{K}{K+1} =$ ————————.

(A) $2\frac{13}{60}$. (B) $3\frac{12}{60}$.

(C) $1\frac{13}{60}$. (D) $2\frac{12}{60}$.

11. If f is continuous at every point of a closed interval I, then f assumes :

(A) An absolute maximum value M but not an absolute minimum value.

(B) An absolute minimum value m but not an absolute maximum value.

(C) Both an absolute maximum value M and an absolute minimum value m.

(D) Neither an absolute maximum nor an absolute minimum.

12. Consider the function $f(x) = \begin{cases} x+1, & -1 \le x < 0 \\ 0, & x = 0 \\ x-1, & 0 < x \le 1 \end{cases}$.

Then which of the following statements is NOT true ?

(A) f is continuous at every point of [–1,1], except at $x = 0$.

(B) f has a non-removable discontinuity at $x = 0$.

(C) f has neither a highest nor a lowest point on [–1,1].

(D) f has the highest value 1 and the lowest value –1 on [–1,1].

13. The only domain points where a function can assume extreme values are ————————.

(A) Critical points and end points. (B) Critical points only.

(C) End points only. (D) None of the above.

Turn over

14. What are the critical points of f when $f'(x) = (x-1)(x-2)$?

(A) 0, 1 and 2. (B) –1 and –2.

(C) 1 and 2. (D) No critical points.

15. The value or values of c that satisfy the equation $\frac{f(b)-f(a)}{b-a} = f'(c)$ in the conclusion of Mean Value Theorem for the function $f(x) = x^2 + 2x - 1$ and the interval [0,1] is :

(A) 1. (B) $\frac{1}{2}$.

(C) $\frac{1}{3}$. (D) $\frac{1}{4}$.

16. The tangent at the point of inflection is called ————————.

(A) Inflectional tangent. (B) Vertical tangent.

(C) Asymptote. (D) None of these.

17. The curve $y = x^4$ has ————————.

(A) Inflection point at $x = 0$. (B) No inflection point at $x = 0$.

(C) No inflection point at $x = 1$. (D) Inflection point at $x = 1$.

18. $\lim_{x \to -\infty} \frac{2x^2 - 3}{7x + 4} =$ ————————.

(A) $\frac{2}{7}$. (B) 0.

(C) ∞. (D) $-\infty$.

19. $y = \sec x = \frac{1}{\cos x}$ has ————————.

(A) Horizontal asymptotes at even-integer multiplies of $\frac{\pi}{2}$.

(B) Vertical asymptotes at even-integer multiplies of $\frac{\pi}{2}$.

(C) Horizontal asymptotes at odd-integer multiplies of $\frac{\pi}{2}$.

(D) Vertical asymptotes at odd-integer multiplies of $\frac{\pi}{2}$.

20. The asymptotes of the curve $y = \frac{x+3}{x+2}$ are ————————.

(A) The line $y = 1$ only. (B) The line $x = -2$ only.

(C) The lines $y = 1$ and $x = -2$. (D) None of these.