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Reg. No..................................

FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2022

Mathematics

MTS 5B 07—NUMERICAL ANALYSIS

(2020 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks. Ceiling is 20.

1. Compute the absolute error and relative error in the approximation of p by p^*, where $p = \pi$, $p^* = \frac{22}{7}$.

2. Write Newton-Raphson Formula and state Intermediate Value theorem.

3. Let $f(x) = e^x - x - 1$. Show that f has a zero of multiplicity 2 at $x = 0$.

4. Show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in $[1, 2]$.

5. Express $\Delta^2 f_0$ and $\Delta^3 f_0$ in terms of the values of the function.

6. State Neville's Method.

7. Write Newton's Divided Difference Formula.

8. What is the degree of accuracy of a quadrature formula ?

9. Write the Legendre Polynomials $P_0(x), P_1(x), P_2(x), P_3(x)$ and $P_4(x)$.

10. Define a convex set in $\mathbb{R}^2$.

11. Give the difference equation form of Runge-Kutta method of order four.

12. Define an implicit three-step method known as the fourth-order Adams-Moulton Technique.

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Section B

Answer any number of questions.
Each question carries 5 marks. Ceiling is 30.

13. Show that $g(x) = \frac{x^2 - 1}{3}$ has a unique fixed point on the interval $[-1, 1]$.

14. The function $f \in C'[a, b]$ has a simple zero at p in (a, b) if and only if $f(p) = 0$ and $f'(p) \neq 0$.

15. Suppose $x_0 = 1, x_1 = 2, x_2 = 3, x_3 = 4, x_4 = 6$ and $f(x) = e^x$. Determine the interpolating polynomial $P_{(1, 2, 4)}$ and use this to approximate $f(5)$.

16. Use the forward difference formula to approximate the derivative of $f(x) = ln\ x$ at $x_0 = 1.8$ with $h = 0.1, 0.05$ and $h = 0.01$ and determine bounds for the approximation errors.

17. Determine value of h that will ensure an approximation error of less than 0.00002 when approximating $\int_0^{\pi} \sin x\, dx$ using composite Simpson's rule.

18. Approximate $\int_{-1}^{1} e^x \cos x\, dx$ using Gaussian quadrature with $n = 3$.

19. Show that the initial value problem

$\frac{dy}{dt} = y - t^2 + 1, 0 \leq t \leq 2, y(0) = 0.5$ is well posed on $D = \{(t, y) : 0 \leq t \leq 2, -\infty < y < \infty\}$.

Section C

Answer any **one** *question.*
The question carries 10 marks.

20. By fixed point iteration method determine a solution accurate to within 10^{-2} for $x^4 - 3x^2 - 3 = 0$ on $[1, 2]$. Use $p_0 = 1$.

21. Apply Taylor's method of order 2 with $N = 10$ to the initial value problem $y' = y - t^2 + 1, 0 \leq t \leq 2, y(0) = 0.5$.

(1 × 10 = 10 marks)