D 103789 **(Pages : 3)** **Name.......................................**

Reg. No...................................

SECOND SEMESTER (CBCSS–UG) DEGREE EXAMINATION, APRIL 2024

Statistics

STA 2C 02—PROBABILITY THEORY

(2019–2023 Admissions)

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical tables are permitted.

Part A (Short Answer Type Questions)

Each question carries 2 marks.
Maximum marks that can be scored from this part is 20.

1. When do you say a random variable is discrete or continuous ?
2. Give the axiomatic definition of probability.
3. State multiplication theorem and addition theorem of probability.
4. If $B \subset A$, show that $P(A \cap B') = P(A) - P(B)$.
5. A continuous random variable X has pdf given by $f(x) = 2x, 0 < x \leq 1$ and 0 elsewhere. Find (i) F (x) ; (ii) P (X $\leq$ 1/2).
6. What are the properties of distribution function ?
7. Examine whether the following is a density function :

$$f(x) = 2x \text{ if } 0 < x \leq 1$$
$$= 4 - 2x \text{ if } 1 < x < 2$$
$$= 0 \text{ elsewhere}$$

8. Define m.g.f. and give any two properties of it.
9. Define characteristic function.
10. Define raw moments and central moments. Give the relationship between them.

Turn over

11. Determine c if $p(x,y) = c(2x+3y)$ where x = 0, 1 and y = 1, 2 is a joint p.d.f. Also find the corresponding distribution function.

12. If $f(x,y) = 1/4$ when $(x,y) \in \{(0,0),(1,0),(0,1),(1,1)\}$ and 0 elsewhere. Examine whether the variables X and Y are independent.

(20 marks)

Part B (Short Essay/Paragraph Type Questions)

Each question carries 5 marks.
Maximum marks that can be scored from this part is 30.

13. Distinguish between mutual independence and pairwise independence of a set events. Give an example to show that pairwise independence does not imply mutual independence.

14. If P_1 and P_2 are probability measures and $0<\lambda<1$, show that $\lambda P_1 + (1-\lambda)P_2 = P$ is a probability measure.

15. Distinguish between probability density function and distribution function. How are the two functions related

16. Let a coin with probability $p, 0 < p < 1$ for turning up of head be tossed until a head appears. Let X denote the number of tails observed. Find P(X = r).

17. Define expectation of a random variable. If X and Y are independent random variables with means 10 and – 5 and variances 4 and 6 respectively. Find a and b such that Z = aX + bY will have mean 0 and variance 28.

18. The pdf of two random variables X and Y is $f(x,y) = 2, 0 \le x \le y \le 1$. Show that E (X) =1/3 and E (Y) = 2/3 and the correlation between X and Y is ½.

19. Define conditional mean and conditional variance in both discrete and continuous case.

(30 marks)

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Part C (Essay Type Questions)

Answer any **one** *question.*
The question carries 10 *marks.*
Maximum marks that can be scored from this part is 10.

20. (a) State and prove Bayes' theorem.

    (b) The probabilities of X,Y and Z become managers are 4 : 2 : 3 .The probabilities that the Bonus scheme will be introduced if X,Y and Z becomes managers are 0.3, 0.5 and 0.8 respectively. If the Bonus scheme was introduced, What is the probability that X is appointed as the manager ?

21. $f(x,y)=\frac{1}{72}(2x+3y), x=0,1,2 \quad y=1,2,3$ is the joint density of (X,Y).

    (a) Find the distribution of X + Y.

    (b) Find the conditional distribution of X given X + Y = 3.

    (c) Examine whether X and Y are independent.

(1 × 10 = 10 marks)