D 10230 (Pages : 3) Name..................................

Reg. No..............................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS-UG)

Mathematics

MAT 5B 05—VECTOR CALCULUS

Time : Three Hours Maximum : 120 Marks

Part A

Answer **all** *questions.*
Each question carries 1 *mark*.

1. Find the domain and range of $z = \sqrt{25 - x^2 - y^2}$.
2. Evaluate $\lim\limits_{(x,y)\to(1,-1)} \dfrac{1+x-y}{2-x+y}$.
3. Define gradient of a scalar function.
4. Compute the divergence of $\vec{f} = xy\vec{i} + yz\vec{j} + xz\vec{k}$.
5. Define solenoidal vector.
6. What do you mean by directional derivative.
7. Write the component test for the differential M(*x, y, z*) *dx* + N(*x, y, z*) *dy* + P(*x, y, z*) *dz* to be exact.
8. Find du if $u = \arcsin \dfrac{x}{y}$.
9. Fill in the blanks : If $\vec{f}$ and $\vec{g}$ are irrotational vector point functions, then $\nabla.\left(\vec{f} \times \vec{g}\right) = \ldots.$
10. State the normal form of Green's theorem in the plane.
11. Fill in the blanks : If $\vec{a}$ is a constant vector and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then of $\nabla\left(\vec{r}.\vec{a}\right) = \ldots.$
12. State Stoke's theorem.

(12 × 1 = 12 marks)

Part B

Answer any **ten** *questions.*
Each question carries 4 *marks*.

13. Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

Turn over

14. Find the vector normal to the surface $\phi(x, y, z) = xyz$ at $(1, -1, 1)$.

15. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at (π, π, π) from $\sin(x + y) + \sin(y + z) + \sin(x + z) = 0$.

16. Prove that $\nabla\left(r^n\right) = nr^{n-2}\vec{r}$.

17. Compute the average value of the function $f(x, y, z) = xyz$ over the boundary of the cube $0 \le x \le 2, 0 \le y \le 2, 0 \le z \le 2$.

18. Evaluate $\int_1^2 \int_3^4 \dfrac{1}{(x+y)^2}\, dx\, dy$.

19. Leaniarize the function $f(x, y, z) = xy + yz + zx$ at (1, 1, 1).

20. Find the directional derivative of $f(x, y, z) = xy$ at (1, 2).

21. Evaluate $\int\int_R (xy)\, dx\, dy$ where R is the positive quadrant of the circle of radius a centred at the origin.

22. Find the flow of $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ along the portion of the circular helix $x = \cos t,\ y = \sin t,\ z = t;\ 0 \le t\ \pi/2$.

23. Test whether $\overline{f} = (yz)\,\overline{i} + (xz)\,\overline{j} + (xy)\,\overline{k}$ is conservative or not.

24. Prove that $div\left(curl\overline{f} = 0\right)$.

25. Verify whether the differential $\left(e^x \cos y + yz\right) dx + \left(xz - e^x \sin y\right) dy\ (xy + z)\, dz$ is exact or not.

26. If S is a closed surface enclosing a volume V then prove that $\int\int_S \overline{culf} \cdot \hat{n} dS = 0$.

(10 × 4 = 40 marks)

Part C

Answer any **six** *question.*
Each question carries 7 marks.

27. Using double integrals prove that $\int_0^\infty e^{-\left(x^2\right)} dx = \sqrt{\pi}/2$.

28. Evaluate the line integral $\int_C ydx + xdy$ where C is the boundary of the square x = 0, x = 1, y = 0 and y = 1.

29. Find the work done by the force field $\vec{f} = 3xy\vec{i} - 58\vec{j} + 10x\vec{k}$ along the space curve $C : \vec{r} = \left(t^2 + 1\right)\vec{i} + 2t^2\vec{j} + t^3\vec{k}$ where $0 \le t \le 2\pi$.

30. Find angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at (2, – 1, 2).

31. Evaluate the volume bounded by $y = x^2, x = y^2$ and the planes $z = 0$ and $z = 3$.

32. Evaluate the area enclosed by the region cut from the plane $x + 2y + 2z = 5$ by the cylinder whose walls are $x = y^2$ and $x = 2 - y^2$.

33. Find the Local extreme values of $f(x, y) = x^2 + y^2 + xy + 3x - 3y + 4$.

34. Evaluate the line integral $\int_C \vec{f} \cdot \vec{dr}$ where C is the boundary of the triangle with vertices (0, 0, 0), (1, 0, 0), (1, 1, 0).

35. Show that $\vec{f} = y \sin z\vec{i} + x \sin z\vec{j} + xy \cos z\vec{k}$ is conservative and find its scalar potential.

(6 × 7 = 42 marks)

Part D

Answer any **two** *question.*
Each question carries 13 *marks.*

36. (a) State Gauss divergence theorem and use it to evaluate the outward flux of $\vec{f} = xy\vec{i} + yz\vec{j} + xz\vec{k}$ through the surface of the cube cut from the first Octant by the planes $x = y = z = 1$.

    (b) Evaluate $\int_{(1,0,0)}^{(0,1,0)} \sin y \cos x dx + \cos y \sin x dy + dz$.

37. Verify Stoke's Theorem for $\vec{f} = \left(x^2 - y^2\right)\vec{i} + 2xy\vec{j}$ over the rectangular region bounded by $x = 0, x = a, y = 0, y = a$.

38. Verify the Tangential form of Green's theorem in the plane for the vector Field $\vec{f} = (x - y)\vec{i} + x\vec{j}$ over the region bounded by the unit circle $x^2 + y^2 = 1$.

(2 × 13 = 26 marks)

D 10230–A **(Pages : 5)** **Name.......................................**

Reg. No...................................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS—UG)

Mathematics

MAT 5B 05 VECTOR CALCULUS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 30 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 5B 05 VECTOR CALCULUS

(Multiple Choice Questions for SDE Candidates)

1. The length of the vector a with initial point p : (3, 2, 5) and terminal point (5, 1, 3) is :

   (A) 3. (B) 4.

   (C) 5. (D) 6.

2. The straight line through the point (1, 3) in the x, y plane and perpendicular to the straight line $x - 2y + 2 = 0$ is :

   (A) $3x - y = 2$. (B) $x + y = 1$.

   (C) $2x + y = 5$. (D) $2x - y = 5$.

3. The volume of the tetrahedron with co-terminal edges representing the vectors $i + j$, $i - j$ and $2k$ is :

   (A) $\frac{2}{3}$. (B) $\frac{3}{2}$.

   (C) $\frac{3}{5}$ (D) $\frac{2}{5}$.

4. The parametric equations for the line through (– 3, 2, – 3) and (1, – 1, 4) are :

   (A) $x = 1 + 4t, y =- 1 - 3t, z = 4 + 7t$. (B) $x = 2 + 4t, y = -2 - 3t, z = -4 + 7t$.

   (C) $x = 3 + 4t, y = 8 - 3t, z = 5 + 7t$. (D) $x = 1 - 4t, y = -1 + 3t, z = -4 - 7t$.

5. The spherical co-ordinate equation for the cone $z = \sqrt{x^2 + y^2}$ is :

   (A) $\Phi = \prod$. (B) $\Phi = \prod/4$.

   (C) $\Phi = \prod/2$. (D) None of these.

6. A particle moves along the curve :

   $x = 3t^2, y = t^2 - 2t, z = t^3$ then the acceleration at $t = 1$ is :

   (A) $6i + 2j + 6k$. (B) $6i + 3k$.

   (C) $6i + 6k$. (D) $6i + 2j + 3k$.

7. The unit tangent vector at a point t to the curve $r = a \cos ti + a \sin t j$:

(A) $-\sin ti - \cos tj$.
(B) $-\sin ti + \cos tj$.
(C) $\cos ti - \sin tj$.
(D) $-\sin ti + \cos tj$.

8. The domain of the function $f(x, y, z) - xyln(z)$:

(A) Entire Space.
(B) $\{(x, y, z) : xyz \neq 0\}$.
(C) Half space $z > 0$.
(D) Half space $z < 0$.

9. Which of the following holds for the function $f(x, y) = \frac{x+y}{x-y}$?

(A) f is continuous everywhere.
(B) f is continuous nowhere.
(C) f is continuous on $\left\{(x, y) \in \mathrm{R}^2 : x \neq y\right\}$.
(D) f is continuous on $\left\{(x' y) \in \mathrm{R}^2 : x = y\right\}$.

10. If $f(x, y) = x \sin xy,$ then the value of $\frac{\partial f}{\partial x}$ at $\left(3, \frac{\pi}{6}\right)$ is :

(A) 0.
(B) 1.
(C) –1.
(D) 2.

11. If $f(x, y) = e^{x+y+1}$ then $\frac{\partial f}{\partial y}$ is :

(A) e^{x+1}.
(B) e^{y}.
(C) e^{x+y+1}.
(D) NOT.

Turn over

12. Let $w(p, v, \delta, \gamma, g) = PV + \frac{\gamma\delta^2}{2g}$ then the partial derivative W_δ is given by :

(A) $\frac{v}{2g}$.

(B) $pv + \frac{vg}{2g}$.

(C) $\frac{vg}{g}$.

(D) NOT.

13. The derivative of $f(x, y) = xe^y + \cos(xy)$ at the point (2, 0) in the direction $A = 3i - 4j$ is :

(A) 1.

(B) 0.

(C) – 1.

(D) 2.

14. The unit normal to the surface $x^2y + 2xz = 4$ at the point (2 – 2, 3) is :

(A) $i + j = k$.

(B) $\frac{2}{3}i + 2j + 5k$.

(C) $\frac{1}{3}i - j - 7k$.

(D) $\frac{1}{3}i - \frac{2}{3}j - \frac{2}{3}k$.

15. The function $f(x, y) = xy$ has a :

(A) Local maximum.

(B) Local minimum.

(C) Both local maximum and minimum.

(D) No local extreme values.

16. The point $p(x, y, z)$ closest to the origin on the plane $2x + y - z - 5 = 0$ is :

(A) $\left(\frac{5}{6}, \frac{-5}{6}, \frac{5}{6}\right)$.

(B) $\left(\frac{5}{3}, \frac{5}{6}, \frac{5}{3}\right)$.

(C) $\left(\frac{5}{3}, \frac{5}{6}, \frac{-5}{6}\right)$.

(D) $\left(\frac{5}{6}, \frac{5}{3}, \frac{-5}{3}\right)$.

17. The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. The points on the Ellipse that lies closest to the origin are :

(A) (1, 0, 0) and (0, 0, 1). (B) (0, 1, 0) and (0, 0, 1).

(C) (1, 0, 0) and (0, 1, 0). (D) (1, 0, 0) and (0, 1, 1).

18. Which among the following is the value of $\int_0^1 \int_0^1 xy\,(x - y)\,dxdy$?

(A) 4. (B) $\frac{2}{3}$.

(C) $\frac{8}{3}$. (D) $\frac{4}{3}$.

19. The volume enclosed by the co-ordinate planes and the portion of the plane $x + y + z = 1$ in the first octant is :

(A) $1/2$. (B) $1/3$.

(C) $1/6$. (D) $1/4$.

20. Which among the following is the value of $\int_1^3 \int_{1/x}^1 \int_0^{\sqrt{xy}} xyz\,dzdydx$?

(A) $\frac{1}{8}(26 + \log 27)$. (B) $\frac{1}{8}(27 - \log 26)$.

(C) $\frac{1}{8}(27 + \log 26)$. (D) $\frac{1}{8}(26 - \log 27)$.