D 12645 **(Pages : 3)** **Name....................................**

Reg. No.................................

FIRST SEMESTER (CBCSS–UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2021 Admissions)

Time : Two Hour and a Half Maximum : 80 Marks

Section A

Answer atleast **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall ceiling 30.

1. Verify that $p \vee p \equiv p$ and $p \wedge p \equiv p$.

2. Let P(x)denote the statement "$x > 3$." What is the truth value of the quantification $\exists x\ \mathrm{P}(x)$, where the universe of discourse is the set of real numbers ?

3. State the barber paradox presented by Bertrand Russell in 1918.

4. Prove that if n is a positive integer, then n is odd if and only if $5n + 6$ is odd.

5. Prove the following formula for the sum of the terms in a "geometric progression" :

$$1 + r + r^2 + \ldots + r^n = \frac{1 - r^{n+1}}{1 - r}.$$

6. Let a and b positive integers such that $a \mid b$ and $b \mid a$. Then prove that $a = b$.

7. Briefly explain Mahavira's puzzle.

8. Find the number of positive integers ≤ 2076 and divisible by neither 4 nor 5.

9. Prove that every composite number n has a prime factor $\leq \lfloor \sqrt{n} \rfloor$.

10. Show that any two consecutive Fibonacci numbers are relatively prime.

Turn over

11. Let a and b be integers, not both zero. Then prove that a and b are relatively prime if and only if there exist integers α and β such that $1 = \alpha a + \beta b$.

12. Prove that if $a|$ and $b|c$, and $(a, b) = 1$, then $ab|c$.

13. Prove that every integer $n \geq 2$ has a prime factor.

14. Let f_n denote the n^{th} Fermat number. Then prove that $f_n = f_{n-1}^2 - 2f_{n-1} + 2$, where $n \geq 1$.

15. Express gcd (28, 12) as a linear combination of 28 and 12.

(10 × 3 = 30 marks)

Section B

Answer atleast **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall ceiling 30.

16. Show that the propositions $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

17. Show that theassertion "All primes are odd" is false.

18. Let b be an integer $\geq 2.$ Suppose $b + 1$ integers are randomly selected. Prove that the difference of two of them is divisible by b.

19. If p is a prime and $p | a_1 a_2 ... a_n,$ then prove that $p | a_i$ at for some i, where $1 \leq i \leq n$.

20. Show that 11 × 14n + 1 is a composite number.

21. There are infinitely many primes of the form 4n + 3.

22. Show that $2^{11213} - 1$ is not divisible by 11.

23. Prove that if $n \geq 1$ and gcd $(a, n) = 1$,then $a^{\phi(n)} \equiv 1 \pmod{n}$.

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) State six standard methods for proving theorems and briefly explain any two of them with the help of examples.

    (b) Using the laws of logic simplify the Boolean Expression $(p \wedge \neg q) \vee q \vee (\neg p \wedge q)$.

25. (a) Prove that there is no polynomial $f(n)$ with integral coefficients that will produce primes for all integers n.

    (b) State the prime number theorem and find six consecutive integers that are composites.

26. (a) State and prove Fundamental Theorem of Arithmetic.

    (b) Find the largest power of 3 that divides 207!

27. (a) Let p be a prime and a any integer such that $p \mid a$. Then show that the least residues of the integers a, $2a$,$3a$,. .. ,$(p - 1)$ a modulo p are a permutation of the integers

    1, 2, 3,...,$(p - 1)$.

    (b) Find the remainder when 24^{1947} is divided by 17.

(2 × 10 = 20 marks)