C 40604 **(Pages : 3)** **Name.......................................**

Reg. No..................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2023

(CBCSS—UG)

Mathematics

MTS 6B 11—COMPLEX ANALYSIS

(2020 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 25 Marks.

1. Define entire function. Give an example.
2. State a necessary condition for analyticity.
3. Prove that $u(x, y) = e^{-x} \sin y$ is harmonic.
4. Prove that $\overline{e^{z}} = e^{\overline{z}}$.
5. Find all values of z satisfying the equation $e^{z-1} = -ie^3$.
6. Find the real and imaginary parts of $\sin(\overline{z})$.
7. Evaluate $\oint_C xydx + x^2 dy$ where C is the curve $y = x^3, -1 \le x \le 2.$.
8. Define simply and multiply connected domains. Give examples for each.
9. State Cauchy's - Goursat theorem and find $\oint_C e^z \, dz$ on a simple closed contour C.
10. Evaluate the integral $\int_{\frac{i}{2}}^{i} e^{\pi z} \, dz$ and write it in the form $a + ib$.

Turn over

11. By using Cauchy's integral formula evaluate $\int_C \frac{z}{z^2+9}\,dz$ where C is the circle $|z-2i|=4$.

12. State root test.

13. Find the Taylor expansion of $f(z)=\frac{1}{1-z}$.

14. Find the Laurent's series expansion of $f(z)=\frac{\cos z}{z}$ in $0<|z|$.

15. Find the pole of $\frac{\sin z}{z^2}$.

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Maximum 35 *Marks.*

16. Prove that if f is differentiable at a point z_0 in a domain D then f is continuous at z_0

17. Find the real constants a, b, c and d so that $f(z)=(3x-y+5)+i(ax+by-3)$ is analytic.

18. Compute the principal value of the complex logarithm Ln z for $z=i$ and $z=1+i$.

19. Find the derivative of the principal value of z^i at the point $z=1+i$.

20. Find the upper bound of the absolute value of $\oint_C \frac{e^z}{z+1}\,dz$ where C is the circle $|z|=4$.

21. Evaluate $\oint_C \frac{1}{\sqrt{z}}\,dz$ where C is the line segment between $z_0=i$ and $z_1=9$.

22. Examine the covergence of the following series on their circle of convergence (a) $\sum_{0}^{\infty} z^n$; and

(b) $\sum_{0}^{\infty} \frac{z^n}{n^2}$.

23. Expand $f(z) = \frac{1}{z(z-1)}$ in a Laurent series valid for $|z| > 1$.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*
Maximum 20 *Marks*

24. (a) State and prove Cauchy's integral formula.

(b) Evaluate $\int_C \frac{z}{z^2+9} dz$ where C is the circle $|z-2i| = 4$.

25. Evaluate $\int_C \frac{dz}{z^2+1}$.

26. (a) State and prove Cauchy's inequality.

(b) State Maximum modulus theorem and find the maximum modulus of $f(z) = 2z + 5i$ on the closed circular region $|z| \leq 2$.

27. State and prove Cauchy's residue theorem and using it evaluate $\int_C \frac{dz}{z^3(z-1)}$ where C is $|z| = 2$.