D 10232 (Pages : 3) Name..................................

Reg. No..............................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS–UG)

Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 120 Marks

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Fill in the blanks : Supremum of the set $S = \{1 - 1/n; n \in N\}$ is ———.
2. Determine the set $A = \{x \in R : |x-1| < |x|\}$.
3. The set of all real numbers which satisfy the inequality $0 \le b < \in, \forall \in > 0$, then b = ———.
4. Fill in the blanks : The Supremum property of R states that ———.
5. State the Trichotomy Property of R.
6. Give the condition for a subset of R to be an interval of R.
7. State the general Arithmetic Geometric mean inequality of real numbers.
8. Fill in the blanks : The characterization theorem of open sets states that ———.
9. State the Bernoulli's inequality.
10. If $a > 0$, then $\lim\left(a^{1/n}\right) =$ ———.
11. Fill in the blanks : Arg (-2π) = ———.
12. Fill in the blanks : The Exponential form of $-1-i$ = ———.

(12 × 1 = 12 marks)

Part B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Define Supremum an Infimum of a set. Find them for the set $S = \left\{1/2^m - 1/3^n; m, n \in N\right\}$.
14. Show that there doesn't exist a rational number r such that $r^2 = 3$.
15. If $a, b \in R$, then prove that $\big||a| - |b|\big| \le |a-b|$.
16. Prove that a real sequence can have at most one limit.

Turn over

17. If $x \in R$ then prove that there exists $n_x \in N$ such that $x < n_x$.

18. Discuss the convergence of the following sequences $X = (x_n)$, defined by (a) $x_n = \left(1 + \frac{1}{n+1}\right)^{n-1}$.

19. Show directly that a bounded monotonic sequence is a Cauchy sequence.

20. Define Cauchy sequence. Test whether $(1/n)$ is a Cauchy sequence or not.

21. Show by an example that intersection of infinitely many open sets in R need not be open.

22. Discuss the convergence of $X = (x_n)$ define by $x_n = n$, if n odd and $x_n = 1/n$, if n even.

23. Show that every bounded sequence of real numbers has a converging subsequence.

24. Test the convergence of the sequence $\left(\frac{\cos n}{n}\right)$.

25. Express the complex number $(\sqrt{3} + i)^7$ in exponential form.

26. Find the principal value of $(-8i)^{\frac{1}{3}}$.

(10 × 4 = 40 marks)

Part C

Answer any **six** *questions.*
Each question carries 7 marks.

27. Prove that the set R of real numbers is uncountable.

28. State and prove the Ratio Test for the convergence of real sequence.

29. Discuss the convergence of the following sequences $X = (x_n)$, defined by (a) $x_n = \left(1 + \frac{1}{n+1}\right)^{n-1}$ and (b) $x_n = \left(\frac{1-2}{n}\right)^n$.

30. X = x_n and Y = y_n be sequences of real numbers converges to x and y respectively, then prove that X . Y converges to xy.

31. (a) Give an example of a convergent sequence (x_n) of positive real numbers with $\lim \left(\frac{x_{n+1}}{x_n}\right) = 1$.

    (b) Give an example of a divergent sequence (x_n) of positive real numbers with $\lim \left(\frac{x_{n+1}}{x_n}\right) = 1$.

(c) Give your comments about the property of the sequence (x_n) of positive real numbers with $\lim\left(\frac{x_{n+1}}{x_n}\right) = 1$.

32. If $X = (x_n)$ is a real sequence and $X_m = (x_{m+n} : n \in N)$ is the m-tail of X ; $m \in N$, then show that X_m converges to x if and only if X converges to x.

33. Let $X = (x_n)$ be a bounded sequence of real numbers and $x \in R$ has the property that "every converging subsequence of $X = (x_n)$ converges to x". Prove that $X = (x_n)$ converges to x.

34. Prove or disprove the following statement : $\|z_1\| - |z_2| \le |(z_1)| - |(z_2)|$, $\forall z_1, z_2 \in C$.

35. Test the convergence of (x_n) defined by $x_n = 1 + 1/2 + 1/3 + \ldots + 1/n$.

(6 × 7 = 42 marks)

Part D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. Show that there exists a positive real number x such that $x^2 = 2$.

37. (a) If $I_n = [a_n, b_n], n \in N$ is a nested sequence of closed and bounded intervals, then prove that there exist a common point in every I_n.

    (b) Test the convergence of (x_n) defined by $x_n = 1 + 1/2! + 1/3! + \ldots + 1/n!$.

38. (a) Define a closed set and "cluster point" of a set. Give examples for each of them.

    (b) Prove that a subset of R is closed in R if and only if it contains all of its cluster points.

(2 × 13 = 26 marks)