D 111972 **(Pages : 3)** **Name.....**

Reg. No.....

THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2024

Mathematics

MTS 3B 03—CALCULUS OF SINGLE VARIABLE—2

(2019—2023 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

All *questions can be attended.*
Each question carries 2 *marks.*

1. Differentiate the function $f(x) = \log\left(\frac{x}{\ln(x)}\right)$.
2. Find the derivative of $y = \log\left(|\sec(x) + \tan(x)|\right)$.
3. Find $\lim_{x\to 0} \frac{1-\cos x}{1-\cos(2x)}$.
4. Show that $\cosh^2 x - \sinh^2 x = 1$.
5. Find $\lim_{n\to\infty} e^{\frac{1+n}{1-n}}$.
6. Determine whether the sequence $a_n = 1 + (-1)^n / n^2$ converges or diverges. If it converges, find the limit.
7. Express $.111\overline{1}\ldots$ as a rational number.
8. State the Squeeze Theorem.
9. By using the power series expansion of sin x, show that $\frac{d}{dx}(\sin x) = \cos x$.
10. Find the Maclaurian series expansion of $\frac{1}{1+x}$.
11. Find the rectangular equation of a curve whose parametric equation is $x = t + 1$, $y = t^2 - 1$.
12. Find the equation of the tangent to the ellipse $x = 3\cos t$, $y = 2\sin t$ at $t = \pi/4$.
13. Find an equation of the line that passes through the point (– 1, 0, 2) and is parallel to the vector $\langle 1, 5, -4\rangle$.

Turn over

14. Find the equation of the surface $z = x^2 + y^2$ in cylindrical co-ordinates.

15. Find $r'(t)$ if $r(t) = 2\cos ti + 3\sin tj + 3tk$.

(15 × 2 = 30 marks)
Max. Ceiling : 25 marks

Section B

All *questions can be attended.*
Each question carries 5 *marks.*

16. Evaluate :

(i) $\lim_{x\to 0} \frac{\tan 5x}{\sin 2x}$.

(ii) $\lim_{x\to 0} \frac{x^3 - 3\sin^2 x}{x^2}$.

17. Find the derivative of $y = (\cos x)^x$.

18. Find $\int x^2 e^{-x} dx$.

19. Use the integral test to determine the series $\sum_1^\infty \frac{1}{n^2}$ converge or diverge.

20. Find the interval of convergence and radius of convergence of the power series $\sum_0^\infty \frac{x^{2n+1}}{(2n+1)!}$.

21. Find the Maclaurian series for $\frac{1}{\sqrt{1-x}}$ and determine its interval of convergence.

22. Find $\frac{d^2y}{dx^2}$ for the parametric equation $x = a\cos t$, $y = b\sin t$.

23. Identify and sketch the graph of the surface $x^2 - x^2 - y^2 = 1$.

(8 × 5 = 40 marks)
Max. Ceiling : 35 marks

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (i) Show that $\int \frac{dx}{\sqrt{4x^2 - 9}} dx = \frac{1}{2}\cosh^{-1}\left(\frac{2x}{3}\right)$, $x > 3/2$.

(ii) Find $\int_{-\infty}^{0} \frac{e^x}{\sqrt{1+e^{2x}}} dx$.

(iii) Find $\lim_{x\to 0} (\tan x)^x$.

25. (i) Let C be the ellipse $r(t) = 3\cos t + 2\sin t$. Find T(t) and N(t) at $t = \pi/4$.

(ii) Find the curvature of the curve $r(t) = ti + \frac{1}{t}j$ at $t = 1$.

26. (i) Find the total arc length of the cardioid $r = 1 - \cos\theta$.

(ii) Find the area of the cardioid $r = 1 + \cos\theta$.

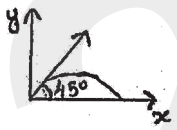
27. A shell fired from a cannon, has a muzzle speed of 80 ft/s. The barrel amkes an angle of 45° with the horizontal and, the barrel opening is assumed to be at ground level.

(a) Find parametric equation for the shell's trajectory.

(b) How high does the shell rise ?

(c) How far does the shell travel horizontally ?

(d) What is the speed of the shell at its point of impact with the ground.



(2 × 10 = 20 marks)