C 20646 **(Pages : 2)** **Name..**

Reg. No....................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 11—COMPLEX ANALYSIS

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Define holomorphic function in a domain D. And give an example for an entire function.
2. Prove or disprove : if f is differentiable a point z_0, then f is continuous at that point.
3. Define harmonic function with example.
4. Prove that $\sin^2 z + \cos^2 z = 1$.
5. State ML inequality.
6. Define the path independence for a contour integral.
7. State maximum modulus theorem.
8. Prove that $\int_a^b f(t)\,dt = -\int_b^a f(t)\,dt$.
9. Prove or disprove if $\lim_{n \to \infty} z_n = 0$, then $\sum_{k=1}^{\infty} z_k$ converges.
10. Find the radius of convergence of $\sum_{k=1}^{\infty} \frac{z^k}{k}$.
11. Define pole of order n. Give an example of a function with simple pole at $z = 1$.
12. Find the principal part in the Laurent series expansion about the origin of the function $f(z) = \frac{\sin z}{z^4}$.

Turn over

13. State Rouche's theorem.

14. Find the residue of $\frac{\sin z}{z}$ at $z = 0$.

15. How many zeroes of are in the disc $|z| = 1$ for the function $f(z) = z^9 - 8z^2 + 5$.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Check whether the function U is harmonic or not if so find its harmonic conjugate $U(x, y) = x^3 - 3xy^2 - 5y$.

17. Find all the solutions of the equation $\sin z = 5$.

18. State and prove Fundamental theorem of algebra.

19. State and prove Morera's theorem.

20. Find the Taylor's series expansion with centre $z_0 = 2i$ of $f(z) = \frac{1}{1-z}$.

21. Identify the singular points and classify them $f(z) = \frac{\sin z e^{\left(\frac{1}{z-1}\right)}}{z(1+z)}$.

22. Find residue of $e^{e^{\left(\frac{1}{z}\right)}}$ at $z = 0$.

23. Find $\int_{-\infty}^{\infty} \frac{1}{x^4+1}\, dx$.

(5 × 6 = 30 marks)

Section C (Essay Questions)

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. State and prove Cauchy Riemann Equation. Also state the sufficient condition for differentiability.

25. State and prove Cauchy's integral formula for derivatives.

26. Expand $f(z) = \frac{1}{z(z-1)}$ in a Laurent series valid for $1 < |z-2| < 2$.

27. State and prove Cauchy's residue theorem.

(2 × 10 = 20 marks)