**D 30177** **(Pages : 4)** **Name.......................................**

**Reg. No...................................**

# FIFTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION NOVEMBER 2022

Mathematics

MAT 5B 05—VECTOR CALCULUS

(2017—2018 Admissions)

Time : Three Hours Maximum : 120 Marks

## Section A

*Answer* **all** *questions.*
*Each question carries* 1 *mark.*

1. The domain of $z = \sqrt{1 - x^2 - y^2}$ is ——————.

2. $\lim\limits_{(x,y)\to(0,0)} \dfrac{y}{2x^2 + 1} =$ ——————.

3. Find $\dfrac{dw}{dt}$ if $w = xy + z,\ x = \cos t,\ y = \sin t,\ z = t$.

4. Find $\dfrac{\partial f}{\partial x}$ if $f(x, y) = \sqrt{x^2 + y^2}$.

5. Define local maximum of a function of two variables.

6. Evaluate $\int_0^1 \int_0^2 xy\,(x - y)\,dxdy$.

7. If R is a simple polar region whose boundaries are the rays $\theta = \alpha$ and $\theta = \beta$ and the curves $r = r_1(\theta)$ and $r = r_2(\theta)$ and if $f(r, \theta)$ is continuous on R, then $\iint_R f(r, \theta)\, dA =$ ——————.

**Turn over**

8. $\int_0^1 \int_0^1 \int_0^1 \left(x^{2'} + y^2 + z^2\right) dz\, dy\, dx.$

9. Define Scalar field.

10. Give a parametrization of the cylinder $x^2 + (y-3)^2 = 9,\ 0 \le z \le 5.$

11. Find curl $\mathbf{F}$ where $\mathbf{F} = x^2 z\mathbf{i} - 2y^3 z^2 \mathbf{j} + xy^2 z\mathbf{k}.$

12. When a vector field is solenoidal ?

(12 × 1 = 12 marks)

## Section B

*Answer any* **ten** *questions.*
*Each question carries* 4 *marks.*

13. Find all first and second order partial derivatives of the function $f(x, y) = x \cos y + ye^x$.

14. Find the linearization of $f(x, y) = x^2 + y^2 + 1$ at the point $(0, 0)$.

15. Evaluate $\iint\limits_D (x+y)\, dxdy$ where D is the domain in the first quadrant of the circle $x^2 + y^2 = 9$

16. Find the tangent plane and normal line of the surface $f(x, y, z) = x^2 + y^2 + z - 9 = 0$ at the point $P_0(1, 2, 4)$.

17. Evaluate $\iiint\limits_V \frac{1}{(x+y+z+1)^3}\, dxdydz$, where V is the volume bounded by the planes $x = 0, y = 0, z = 0$ and $x + y + z = 1$.

18. If $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $|\mathbf{r}| = r$, then show that $\nabla\left(\frac{1}{r}\right) = -\frac{\mathbf{r}}{r^3}$.

19. Find the work done in moving a particle once round a circle C in the $xy$-plane : the circle has centre at the origin at radius 3 and the force field is given by $\mathbf{F} = (2x - y + z)\,\mathbf{i} + \left(x + y - z^2\right)\mathbf{j} + (3x - 2y + 4z)\mathbf{k}$.

20. Find the work done by the conservative field $\mathbf{F} = xz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k} = \nabla(xyz)$ along any smooth curve C joining the points $(-1, 3, 9)$ to $(1, 6, -4)$.

21. Using Green's theorem, evaluate the integral $\oint_C xydy - y^2 dx$,

where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$.

22. Find *unit normal* to the surface $x^2 y + 2xz = 4$ at the point $(2, -2, 3)$.

23. If $v = f\left(\frac{x}{z}, \frac{y}{z}\right)$, show that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} + z\frac{\partial v}{\partial z} = 0$.

24. Find the Centroid of the solid (with density given by $\delta = 1$) enclosed by the cylinder $x^2 + y^2 = 4$, bounded above by the paraboloid $z = x^2 + y^2$ and below by the $xy$-plane.

25. Integrate $G(x, y, z) = x^2$ over the cone $z = \sqrt{x^2 + y^2}, 0 \le z \le 1$.

26. Use Stokes's theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, if $\mathbf{F} = xz\mathbf{i} + xy\mathbf{j} + 3xz\mathbf{k}$ and C is the boundary of the portion of the plane $2x + y + z = 2$ in the first octant traversed counterclockwise as viewed from above.

(10 × 4 = 40 marks)

## Section C

*Answer any* **six** *questions.*
*Each question carries 7 marks.*

27. Show that $f(x, y) = \begin{cases} \frac{4x^2 y}{x^3 + y^3}, & (x, y) \ne (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$

is continuous at every point except the origin.

28. Find the local extreme values of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.

29. Find the volume of the upper region D cut from solid sphere $\rho \leq 1$ by the cone $\phi = \pi/3$.

30. Evaluate $\int_0^4 \int_{x=y/2}^{x=(y/2)+1} \frac{2x-y}{2} \, dxdy$ by applying the transformation $u = \frac{2x-y}{2}, v = \frac{y}{2}$ and integrating over an appropriate region in the $uv$-plane.

31. Integrate $f(x, y, z) = x - 3y^2 + z$ over the line segment C joining the origin and the point.

32. Show that $ydx + xdy + 4dz$ is exact, and evaluate the integral $\int_{(1,1,1)}^{(2,3,-1)} ydx + xdy + 4dz$ over the line segment from $(1,1,1)$ to $(2,3,-1)$.

33. Using Green's theorem in the plane for $\oint_C (xy\, dx + x^2\, dy)$, where C is the curve enclosing the region bounded by the parabola $y = x^2$ and the line $y = x$.

34. Find the area of the cap cut from the hemisphere $x^2 + y^2 + z^2 = 2, z \geq 0$, by the cylinder $x^2 + y^2 = 1$.

35. Evaluate the integral $I = \int_C (3x^2 dx + 2yzdy + y^2\, dz)$ from $A:(0,1,2)$ to $B:(1,-1,7)$ by showing that **F** has a potential.

(6 × 7 = 42 marks)

## Section D

*Answer any* **two** *questions.*
*Each question carries* 13 *marks.*

36. If $u = f(r)$ and $x = r\cos\theta, y = r\sin\theta$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

37. The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the point on the ellipses that lie closest to and farthest from the origin.

38. Verify the Divergence Theorem for the field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$.

(2 × 13 = 26 marks)