D 30569 (Pages : 3) Name.......................................

Reg. No...................................

FIFTH SEMESTER (CBCSS–UG) DEGREE EXAMINATION NOVEMBER 2022

Mathematics

MTS 5B 06—BASIC ANALYSIS

(2020 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 Marks.
Maximum 25 Marks.

1. Define denumerable set. Give an example.
2. If $a \in \mathbb{R}$, then prove that $a \cdot 0 = 0$.
3. Let a, b, c be elements of $\mathbb{R}$ and if $a > b$ and $b > c$, then prove that $a > c$.
4. Prove that $|-a| = |a|$ for all $a \in \mathbb{R}$.
5. Describe Fibonacci sequence.
6. State Monotone Convergence Theorem.
7. Define Cauchy sequence. Give an example.
8. Define properly divergent sequence.
9. Show that $\mathbb{R} = (-\infty, \infty)$ is open.
10. Describe any two properties of Cantor Set.
11. Whether the sequence $\left(1, \frac{1}{2}, 3, \frac{1}{4}, \ldots\right)$ is convergent ? Justify your answer.
12. Find the principal cube root at the point $z = i$.
13. Define bounded subset of the complex plane.

Turn over

14. Find the reciprocal of $z = 2 - 3i$.

15. Express $-\sqrt{3} - i$ in polar form.

Section B

Answer any number of questions.
Each question carries 5 *Marks.*
Maximum 35 *marks.*

16. State and prove Cantor's Theorem.

17. Prove that there does not exist a rational number r such that $r^2 = 2$.

18. (a) Define supremum of a set of real numbers.

    (b) Prove that there can be only one supremum of a given subset S of $\mathbb{R}$, if it exists.

19. Prove that $\lim\left(\frac{1}{n^2}\right) = 0$.

20. If $0 < b < 1$, then prove that $\lim(b^n) = 0$.

21. Prove that the intersection of an arbitrary collection of closed sets in $\mathbb{R}$ is closed.

22. Show that the complex function $f(z) = z + 3i$ is one-to-one on the entire complex plane and find a formula for its inverse function.

23. If $f(z) = \frac{z}{\bar{z}}$ then show by two path test that $\lim\limits_{z \to 0} f(z)$ does not exist.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. State and prove Monotone Subsequence Theorem.

25. Prove that a monotone sequence of real numbers is convergent if and only if it is bounded. If $X = (x_n)$ is a bounded increasing sequence, then prove that :

    $\lim(x_n) = \sup\{x_n : n \in \mathrm{N}\}$.

26. Let X = (x_n) be a sequence of real numbers that converges to x and suppose that $x_n \geq 0$. Then prove that the sequence $\left(\sqrt{x_n}\right)$ converges and $\lim\left(\sqrt{x_n}\right) = \sqrt{x}$.

27. Show that the function f defined by :

$$f\,(z) = \sqrt{r}e^{i\theta/2}, - \pi < \theta < \pi$$

is a branch of the multiple-valued function $\mathrm{F}\,(z) = z^{1/2}$.