**D 10670** **(Pages : 3)** **Name..........................................**

**Reg. No.....................................**

# FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5B 09—INTRODUCTION TO GEOMETRY

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

## Section A

*Answer at least* **eight** *questions.*
*Each question carries* 3 *marks.*
*All questions can be attended.*
*Overall Ceiling* 24.

1. Find the equation of the tangent at the point with parameter $t$ to the parabola with parametric equations $x = at^2, y = 2at$ where $t \in \mathbb{R}$.

2. Let E be a parabola with parametric equations $x = t^2, y = t, t \in \mathbb{R}$. Find focus, vertex axis and directrix of E.

3. Prove that the equation of the tangent at the point $(x_1, y_1)$ to an ellipse is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$.

4. Write the equation of the conic $x^2 - 4xy + 4y^2 - 6x - 8y + 5 = 0$ in matrix form.

5. Show that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal for each real number $\theta$.

6. Let the Euclidean transformations $t_1$ and $t_2$ of $\mathbb{R}^2$ be given by :

$$t_1(\mathrm{X}) = \begin{bmatrix} \frac{3}{5} & \frac{-4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix} \mathrm{X} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} \text{ and}$$

$$t_2(\mathrm{X}) = \begin{bmatrix} \frac{-4}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{bmatrix} x + \begin{bmatrix} -2 \\ 1 \end{bmatrix}. \text{ Find } t_2 \circ t_1.$$

**Turn over**

7. Find the inverse of the affine transformation $t(\mathrm{X}) = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} \mathrm{X} + \begin{bmatrix} 4 \\ -2 \end{bmatrix}$.

8. State fundamental theorem of affine geometry.

9. Prove that an affine transformation maps straight lines to straight lines.

10. State Desargue's theorem.

11. Find the equation of the line that passes through the point $[1, 2, 3]$ and $[2, -1, 4]$.

12. Find the point of intersection of the lines in $\mathbb{RP}^2$ with equations $x + 6y - 5z = 0$ and $x - 2y + z = 0$.

(8 × 3 = 24 marks)

**Section B**

*Answer at least* **five** *questions.*
*Each question carries* 5 *marks.*
*All questions can be attended.*
*Overall Ceiling* 25.

13. Let PQ be an arbitrary chord of the ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Let M be the midpoint of PQ. Prove that the following expression is independent of the choice of P and Q : Slope of OM × Slope of PQ.

14. State and prove reflection properties of the ellipse.

15. Prove that the set of all affine transformations A(2) forms a group under the operation of composition of functions.

16. Determine the image of the line $y = 2x$ under the affine transformation $t(\mathrm{X}) = \begin{pmatrix} 4 & 1 \\ 2 & 1 \end{pmatrix} \mathrm{X} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \mathrm{X} \in \mathbb{R}^2$.

17. Determine the affine transformation which maps the points $(2, 3), (1, 6)$ and $(3, -1)$ to the points $(1, -2), (2, 1)$ and $(-3, 5)$ respectively.

18. Prove that affine transformations maps ellipses to ellipses, parabolas to parabolas and hyperbolas to hyperbolas.

19. Determine the point of $\mathbb{RP}^2$ at which the line through the points $[1, 2, -3]$ and $[2, -1, 0]$ meets the line through the points $[1, 0, -1]$ and $[1, 1, 1]$.

(5 × 5 = 25 marks)

## Section C

*Answer any* **one** *question.*
*The question carries* 11 *marks.*

20. Prove that the conic E with equation $3x^2 - 10xy + 3y^2 + 14x - 2y + 3 = 0$ is a hyperbola. Determine its centre, and its major and minor axes.

21. State and prove Ceva's theorem.

(1 × 11 = 11 marks)