D 10232 **(Pages : 3)** **Name..................................**

Reg. No...............................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS–UG)

Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

Time : Three Hours Maximum : 120 Marks

Part A

Answer **all** *questions.*
Each question carries 1 *mark*.

1. Fill in the blanks : Supremum of the set $S = \{1 - 1/n; n \in N\}$ is ————.
2. Determine the set $A = \{x \in R : |x - 1| < |x|\}$.
3. The set of all real numbers which satisfy the inequality $0 \le b < \in, \forall \in > 0,$ then b = ————.
4. Fill in the blanks : The Supremum property of R states that ————.
5. State the Trichotomy Property of R.
6. Give the condition for a subset of R to be an interval of R.
7. State the general Arithmetic Geometric mean inequality of real numbers.
8. Fill in the blanks : The characterization theorem of open sets states that ————.
9. State the Bernoulli's inequality.
10. If $a > 0$, then $\lim(a^{1/n}) =$ ————.
11. Fill in the blanks : Arg (-2π) = ————.
12. Fill in the blanks : The Exponential form of $-1 - i$ = ————.

(12 × 1 = 12 marks)

Part B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Define Supremum an Infimum of a set. Find them for the set $S = \{1/2^m - 1/3^n; m, n \in N\}$.
14. Show that there doesn't exist a rational number r such that $r^2 = 3$.
15. If $a, b \in R,$ then prove that $||a| - |b|| \le |a - b|$.
16. Prove that a real sequence can have at most one limit.

Turn over

17. If $x \in \mathrm{R}$ then prove that there exists $n_x \in \mathrm{N}$ such that $x < n_x$.

18. Discuss the convergence of the following sequences $\mathrm{X} = (x_n)$, defined by (a) $x_n = \left(1 + \frac{1}{n+1}\right)^{n-1}$.

19. Show directly that a bounded monotonic sequence is a Cauchy sequence.

20. Define Cauchy sequence. Test whether $(1/n)$ is a Cauchy sequence or not.

21. Show by an example that intersection of infinitely many open sets in R need not be open.

22. Discuss the convergence of $\mathrm{X} = (x_n)$ define by $x_n = n$, if n odd and $x_n = 1/n$, if n even.

23. Show that every bounded sequence of real numbers has a converging subsequence.

24. Test the convergence of the sequence $\left(\frac{\cos n}{n}\right)$.

25. Express the complex number $(\sqrt{3} + i)^7$ in exponential form.

26. Find the principal value of $(-8i)^{\frac{1}{3}}$.

(10 × 4 = 40 marks)

Part C

Answer any **six** *questions.*
Each question carries 7 marks.

27. Prove that the set R of real numbers is uncountable.

28. State and prove the Ratio Test for the convergence of real sequence.

29. Discuss the convergence of the following sequences $\mathrm{X} = (x_n)$, defined by (a) $x_n = \left(1 + \frac{1}{n+1}\right)^{n-1}$ and (b) $x_n = \left(\frac{1-2}{n}\right)^n$.

30. X = x_n and Y = y_n be sequences of real numbers converges to x and y respectively, then prove that X . Y converges to xy.

31. (a) Give an example of a convergent sequence (x_n) of positive real numbers with $\lim \left(\frac{x_{n+1}}{x_n}\right) = 1$.

    (b) Give an example of a divergent sequence (x_n) of positive real numbers with $\lim \left(\frac{x_{n+1}}{x_n}\right) = 1$.

(c) Give your comments about the property of the sequence (x_n) of positive real numbers with $\lim\left(\frac{x_{n+1}}{x_n}\right)=1$.

32. If $X=(x_n)$ is a real sequence and $X_m=(x_{m+n}:n\in N)$ is the m-tail of X ; $m\in N$, then show that X_m converges to x if and only if X converges to x.

33. Let $X=(x_n)$ be a bounded sequence of real numbers and $x\in R$ has the property that "every converging subsequence of $X=(x_n)$ converges to x". Prove that $X=(x_n)$ converges to x.

34. Prove or disprove the following statement : $\|z_1\|-|z_2|\leq|(z_1)|-|(z_2)|,\ \forall z_1,z_2\in C$.

35. Test the convergence of (x_n) defined by $x_n=1+1/2+1/3+\ldots+1/n$.

(6 × 7 = 42 marks)

Part D

Answer any **two** *questions.*
Each question carries 13 *marks.*

36. Show that there exists a positive real number x such that $x^2 = 2$.

37. (a) If $I_n=[a_n,b_n],\ n\in N$ is a nested sequence of closed and bounded intervals, then prove that there exist a common point in every I_n.

    (b) Test the convergence of (x_n) defined by $x_n=1+1/2!+1/3!+\ldots+1/n!$.

38. (a) Define a closed set and "cluster point" of a set. Give examples for each of them.

    (b) Prove that a subset of R is closed in R if and only if it contains all of its cluster points.

(2 × 13 = 26 marks)

D 10232-A **(Pages : 4)** **Name.......................................**

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Mathematics

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 30 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MAT 5B 07—BASIC MATHEMATICAL ANALYSIS

(Multiple Choice Questions for SDE Candidates)

1. If $A_n = \{n, n+1, n+2, \ldots\}$, then $\bigcap_{n=1}^{\infty} A_n =$ ——————.

(A) 1. (B) ϕ.

(C) ∞. (D) n.

2. Consider the function $f(x) = \dfrac{1}{x^2}, x \neq 0.$ Determine the image $f(E)$ where $E = \{x \in R : 1 \leq x \leq 2\}$.

(A) $[1/4, 1]$. (B) $[1/2, 1]$.

(C) $(0, 1/4]$. (D) $[0, 1/4]$.

3. If $f(x) = 2x$, and $g(x) = 3x^2 - 1$, then $(f \circ g)(x)$ is ——————.

(A) $12x^2 - 1$. (B) $12x^2 - 2$.

(C) $6x^2 - 1$. (D) $6x^2 - 2$.

4. Which of the following is true ?

(A) $(f \circ g)^{-1}(H) = g^{-1}\left(f^{-1}(H)\right)$. (B) $(f \circ g)^{-1}(H) = f^{-1}\left(g^{-1}(H)\right)$.

(C) $(f \circ g)^{-1}(H) \subseteq g^{-1}\left(f^{-1}(H)\right)$. (D) $(f \circ g)^{-1}(H) \subseteq f^{-1}\left(g^{-1}(H)\right)$.

5. For each $n \in N$ let $A_n = \{(n+1)k : k \in N\}$. Then, $A_1 \cap A_2$ is :

(A) A_3. (B) A_4.

(C) A_5. (D) A_6.

6. The function $f : \mathbb{R} \to (-1, 1)$ defined by $f(x) = x / \sqrt{x^2 + 1}$ is :

(A) A surjection but not injection. (B) An injection but not surjection.

(C) Neither injection nor surjection. (D) A bijection.

7. Which of the following set is not countable ?

   (A) {1, 2, ..., n}
   (B) The set $\mathbb{N}$ of natural numbers.
   (C) The set $\mathbb{Q}$ of rational numbers.
   (D) The interval (0,1).

8. If S = {2, 3, 4} the number of elements in P (S), the power set of S, is :

   (A) 3.
   (B) 6.
   (C) 8.
   (D) 9.

9. Which of the following is *not* true ?

   (A) If $a > b$ and $c > 0$, then $a + c > b + c$.
   (B) If $a > b$ and $c < 0$, then $a + c < b + c$.
   (C) If $a > b$ and $c > 0$, then $ac > bc$.
   (D) If $a > b$ and $c < 0$, then $ac <$ bc.

10. If $a \in \mathbb{R}$ such that, $0 \leq a < \varepsilon$ for every $\varepsilon > 0$ then, :

   (A) $a > 0$.
   (B) $a \neq 0$.
   (C) $a = 0$.
   (D) None of these.

11. Let $S = \{x \in \mathbb{R} : x < 2\}$. Then :

   (A) Neither sup S nor inf S exist.
   (B) Both sup S and inf S exist.
   (C) Sup S exists but less than 2.
   (D) Sup S equal to 2.

12. If $S = \left\{1 - \frac{(-1)^n}{n}, n \in \mathbb{N}\right\}$, then :

   (A) Sup S = 2, Inf S = 1/2.
   (B) Sup S = 2, Inf S = 0.
   (C) Sup S = 1, Inf S = 1/2.
   (D) Sup S = 1, Inf S = 0.

13. The binary representation of 3/8 is :

   (A) 0. 0111111 . . . .
   (B) 0. 0101000 . . . .
   (C) 0. 1011111 . . . .
   (D) 0.0101111 . . . .

14. The rational number represented by 7.3141414 ———— is :

(A) 7245/990. (B) 7249/990.

(C) 7241/990. (D) 7243/990.

15. The sixth term of the Fibonacci sequence is ————.

(A) 5. (B) 6.

(C) 8. (D) 13.

16. Limit of the sequence $\left(\frac{3n+2}{2n+1}\right)$ is ————.

(A) 3. (B) 1/2.

(C) 2. (D) 3/2.

17. The smallest value of $K(\varepsilon)$ corresponding to $\varepsilon = .01$ for the sequence $\left(\frac{1}{n}\right)$ is ————.

(A) 10. (B) 50.

(C) 100. (D) 101.

18. Which of the following is false ?

(A) If (x_n) is a convergent sequence then $\left(x_n^2\right)$ is convergent.

(B) If (x_n) is a convergent sequence, and $x_n \geq 0$ for every n, then $\left(\sqrt{x_n}\right)$ is convergent.

(C) If $\left(x_n^2\right)$ is a convergent sequence then (x_n) is convergent.

(D) If (x_n) is a convergent sequence then $\left(x_n^3\right)$ is convergent.

19. The limit of the sequence $\left(1+\frac{1}{2n}\right)^n$ is :

(A) 1. (B) ∞.

(C) e. (D) $\sqrt{e}$.

20. The sequence $\left(4, -2, 0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \ldots \ldots\right)$

(A) Monotone decreasing. (B) Monotone increasing.

(C) Ultimately Monotone decreasing. (D) Ultimately Monotone increasing.