D 31615 **(Pages : 3)** **Name.......................................**

Reg. No...................................

THIRD SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2022

Mathematics

MAT 3B 03—CALCULUS AND ANALYTIC GEOMETRY

(2017–2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A

(Objective Type. Answer **all twelve** *questions.*

1. Define natural logarithm function.
2. Define a sequence.
3. Define a non-decreasing sequence.
4. Find a formula for nth term of the sequence 1, 6, 11, 16, 21, ————.
5. Define sinh (x).
6. Define parabola.
7. Fill in the blanks : The eccentricity of a parabola is e = ————.
8. Write a parametrization of the circle $x^2 + y^2 = 4$.
9. $\lim_{n \to \infty} \sqrt[n]{n} =$ ————.
10. Fill in the blanks : If the point $P_0(r_0, \theta_0)$ n is the foot of the perpendicular from the origin to the line L and $r \geq 0,$ the equation of the line L in polar form is r = ————.
11. The polar equation $r = \dfrac{k}{1 + \cos\theta}$ represents a conic whose eccentricity is ————.
12. Maclaurins series for the function e^z is ————.

(12 × 1 = 12 marks)

Turn over

Part B

(Short Answer Type. Answer any **nine** *questions).*

13. Find $\int_0^{\ln 2} 4e^x \sinh x dx$.

14. Find k if $e^{-2k} = 10$.

15. Show that e^x grows faster than x^2 as x tends to ∞.

16. Show that $\lim_{n \to \infty} \frac{1}{n^2} = 0$.

17. If the sequence $\{a_n\}$ is recursively defined as $a_n = na_{n-1}$ and $a_1 = 1$, find a_6.

18. Find $\lim_{x \to \infty} x^{\frac{1}{x}}$.

19. Find the eccentricity of the hyperbola $x^2 - y^2 = 1$.

20. Find the focus and directrix of the parabola $x^2 = 100y$.

21. Determine the conic section from the equation $xy - y^2 - 4y + 1 = 0$.

22. Replace the polar equation $r^2 = 4r \cos \theta$ by equivalent Cartesian equation.

23. Graph the sets of points whose polar co-ordinates satisfy the conditions $-3 \le r \le 3$ and $\theta = \pi/2$.

24. Find the equation for the ellipse with eccentricity 1/2 and directrix $x = 1$.

(9 × 2 = 18 marks)

Part C

(Short Essay Type. Answer any **six** *questions).*

25. Show that the sequence $\left\{\left(\frac{n+1}{n-1}\right)^n\right\}$ converge and find the limit.

26. Find a formula for the n^{th} partial sum of the series $\frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \dots + \frac{1}{(n+1)(n+2)} + \dots$ and use it to find the series sum if it converges.

27. Show that $\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots, -1 < x < 1.$

28. If $\sum a_n$ converges, show that $\lim a_n = 0$.

29. Find the surface area generated by revolving the curves $x = \cos t$, $y = 2 + \sin t$, $0 \leq t \leq 2\pi$ about x-axis.

30. Find the sum of the series : $\frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \ldots$

31. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges or diverges.

32. Check whether $\sum_{n=2}^{\infty} \frac{1+n\ln n}{n^2+5}$ converges or diverges.

33. Find the tangent to the right-handed hyperbola branch $x = \sec t$, $y = \tan t$, $-\pi/2 < t < \pi/2$.

(6 × 5 = 30 marks)

Part D

(Essay Type. Answer any **two** *questions).*

34. (a) Graph the curve $r^2 = \sin 2\theta$.

    (b) Find the length of the astroid $x = \cos 3t$, $y = \sin 3t$, $t \in [0, 2\pi]$.

35. (a) Show that $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ converges.

    (b) Show that $\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right), x \geq 1.$

36. Find the area inside the smaller loop of the limacon $r = s \cos \theta + 1$.

(2 × 10 = 20 marks)