D 30560 **(Pages : 3)** **Name.....**

Reg. No.....

FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2022

Mathematics

MTS 5B 06—BASIC ANALYSIS

(2019 Admissions only)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

All questions can be answered.
Each question carries 2 *marks. Ceiling is* 25.

1. Verify whether the set $\mathbb{Z}$ of all integers is denumerable ?
2. State Cantor's theorem ?
3. Let $a, b, c \in \mathbb{R}$. Then if $a > b$ and $c > 0$, then show that $ac > bc$.
4. Determine the set of all real numbers x such that $2x + 3 \leq 6$.
5. Prove that if x is a rational number and y is an irrational number, then $x + y$ is an irrational number ?
6. State the completeness property of $\mathbb{R}$.
7. Find the Infimum and Supremum if it exists of $A = \{x \in \mathbb{R} : 2x + 5 > 0\}$.
8. State Archimedean property ?
9. A bounded sequence is always convergent. Prove or disprove ?
10. Test the convergence of $\left((-1)^n\right)$.
11. Find the reciprocal of $z = 2 - 3i$.
12. What can be said about the complex number z if $z = \bar{z}$.

Turn over

13. Find the values of the complex exponential function e^z at $z = 2 + \pi i$.

14. Is $\left(\left(1+(-1)^n\right)\right)$ a Cauchy sequence ?

15. Express $1 + i$ in polar form.

Section B

All questions can be answered.
Each question carries 5 *marks. Ceiling is* 35.

16. Show that the set of rational numbers is denumerable ?

17. If $a, b \in \mathbb{R}$, then prove that $\big|\,|a|-|b|\,\big| \le |a-b|$.

18. State and prove Bernoulli's inequality ?

19. State and prove density theorem ?

20. Let S be a non empty bounded set in $\mathbb{R}$. If $b < 0$ and $bS = \{bs : s \in S\}$ then prove that $\ln f\,(bS) = b\,\text{Sup}\,S$.

21. Show that every convergent sequence is bounded.

22. Find the image of the vertical line $z = 1$ under the complex mapping $W = z^2$.

23. Find the three cube roots of $Z = i$.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Let $a, b, c \in \mathbb{R}$. Then if $ab > 0$ then show that either $a > 0$ and $b > 0$ or $a < 0$ and $b < 0$.

(b) Let $a, b, c \in \mathbb{R}$. Then if $ab < 0$ then show that either $a > 0$ and $b < 0$ or $a < 0$ and $b > 0$.

(c) If $1 < C$, then show that $1 < C < C^2$.

25. (a) Prove that a Cauchy sequence of real numbers is bounded.

(b) If $C > 0$ then show that $\lim \left(C^{\frac{1}{n}} \right) = 1.$

26. (a) The polynomial equation $x^3 - 7x + 2 = 0$ has a solution between 0 and 1. Use an approximate contractive sequence to calculate the solution correct to 4 decimal places

(b) Show that $\lim \left(n^{\frac{1}{n}} \right) = 1.$

27. (a) Find the product and quotient of the two complex numbers $z_1 = i$ and $z_2 = -\sqrt{3} - i.$

(b) Find the set of all points in the complex plane that satisfy $|z| = |z - i|.$

(c) Find the image of the horizontal line $y = 3$ under the mapping $f(z) = \overline{z}.$

(2 × 10 = 20 marks)