D 12034 **(Pages : 3)** **Name.....**

Reg. No.....

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2021

Mathematics

MTS 3C 03—MATHEMATICS – 3

(2019–2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Evaluate $\int_0^1 \left(t\hat{i} + 3t^2\hat{j} + 4t^3\hat{k}\right)dt.$

2. The position of a moving particle is $\bar{r}(t) = t^2\hat{i} + t\hat{j} + t^3\hat{k}.$ Find velocity and acceleration of the particle at $t = 2$.

3. If $z = e^{-y}\cos x$ find $\frac{\partial^2 z}{\partial x\,\partial y}$.

4. Find the level surface of $F(x, y, z) = x^2 + y^2 + z^2$ passing through (1, 1, 1).

5. Evaluate $\oint_C x\,dx,$ where C is the circle $x = \cos t, y = \sin t, 0 \le t \le 2\pi.$

6. Show that $\text{curl}\,\vec{r} = \vec{0}$.

7. State Green's theorem in the plane.

8. Evaluate $\int_0^3 \int_0^2 \int_0^1 xyz\,dx\,dy\,dz.$

9. Write the equation of the circle with centre (1, 2) and radius 4 in the complex plane.

Turn over

10. Find the value of i^{2i}.

11. Evaluate $\oint_C \frac{Z e^z}{(z-3)} dz$, where C is $|z| = 2$.

12. Evaluate $\oint_C \frac{dz}{z}$, where C is $|z| = 1$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Use chain rule to find $\frac{dw}{dx}$ at (0,1, 2) for $w = xy + yz$; $x = \cos x$, $y \sin x$, $z = e^x$.

14. Find the directional derivative of $f(x, y) = \sqrt{x^2 y + 2y^2 z}$ at $(-2, 2, 1)$ in the direction of the negative z-axis.

15. Find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$ using double integrals.

16. Use polar coordinates to evaluate $\int_0^2 \int_x^{\sqrt{8-x^2}} \frac{1}{5 + x^2 + y^2} dy\, dx$.

17. Show that $f(z) = \left(2x^2 + y\right) + i\left(y^2 - x\right)$ is not analytic at any point.

18. Evaluate $\oint_C \frac{5z + 7}{z^2 + 2z - 3} dz$, where C is the circle $|z - 2| = 2$.

19. Evaluate $\int \mathrm{Re}\, z\, dz$ along a line segment from $z = 0$ to $z = 1 + 2i$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Let $\vec{F}(x,y,z) = z\hat{j} + z\hat{k}$ represents the flow of a liquid. Find the flux of $\vec{F}$ through the surface S given by that portion of the plane $z = 6 - 3x - 2y$ in the first octant oriented upward.

21. Use triple integrals to find the volume of the solid with in the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$.

(1 × 11 = 11 marks)