C 4387 **(Pages : 3)** **Name.......................................**

Reg. No..................................

SECOND SEMESTER (CBCSS–UG) DEGREE EXAMINATION, APRIL 2021

Mathematics

MTS 2B 02—CALCULUS OF SINGLE VARIABLE – I

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Find two functions f and g such that $\mathrm{F} = g \circ f$ if $\mathrm{F}(x) = (x+2)^4$.

2. Let $f(x) = \begin{cases} -x+3 & \text{if } x < 2 \\ \sqrt{x-2+1} & \text{if } x \geq 2 \end{cases}$.

   Find $\lim\limits_{x \to 2} f(x)$ if it exists.

3. Find the values of x for which the function $f(x) = x^8 - 3x^4 + x + 4 + \dfrac{x+1}{(x+1)(x-2)}$ is continuous.

4. Find $\lim\limits_{x \to o} \dfrac{\tan x}{x}$.

5. Find the instantaneous rate of change of $f(x) = 2x^2 + 1$ at $x = 1$.

6. If $f(x) = 2x^3 - 4x$. Find $f'(-2)$ and $f'(0)$.

7. Find the extreme values $f(x) = 3x^4 - 4x^3 - 8$ on $[-1, 2]$.

8. Determine where the graph of $f(x) = x^3 - 6x$ is concave upward and where it is concave downward.

9. Find $\lim\limits_{x \to -1} \dfrac{1}{x+1}$.

10. Find the horizontal asymptote of the graph of $f(x) = \dfrac{1}{x-1}$.

Turn over

11. Find $\int \frac{\sin t}{\cos^2 t}\, dt.$

12. Find $\int \frac{1}{x \log x}\, dx.$

13. Given that $\int_{-2}^{2} f(x)\, dx = 3$ and $\int_{0}^{2} f(x)\, dx = 2$, evaluate $\int_{2}^{0} f(x)\, dx$.

14. Find the area of the region bounded by the graphs of $y = 2 - x^2$ and $y = -x$.

15. Find the volume of the solid obtained by revolving the region bounded by $y = x^3$, $y = 8$ and $x = 0$ about the y-axis.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Show that the function $f(x) = |\ x\ |$ is differentiable everywhere except at 0.

17. Show that if the function f is differentiable at a, then f is continuous at a.

18. Verify that the function $f(x) = x^2 + 1$ satisfies the hypothesis of the mean value theorem on [0, 2] and find all values of c that satisfy the conclusion of the theorem.

19. Find the relation extrema if any of the function $h(t) = \frac{1}{3}t^3 - 2t^2 - 5t - 10.$

20. The velocity function of a car moving along a straight road is given by $v(t) = t - 20$, for $0 \le t \le 40$, where $r(t)$ is measured in feet per second and t in seconds. Show that at $t = 40$, the car will be in the same position as it was initially.

21. (a) State mean value theorem for integrals.

    (b) Verify mean value theorem for $f(x) = x^2$ on [1, 4].

22. (a) Use differentials to obtain an approximation of the arc length of the graph of $y = 2x^2 + x$ from P (1,3) to Q (1.1, 3.52).

    (b) Find the work done in lifting a 50 – lb sack of potatoes to a weight of 4 ft above the ground.

23. Find the length of the graph of $f(x) = \frac{1}{3}x^3 + \frac{1}{4x}$ on the interval [1, 3].

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Find the slope and an equation of the tangent line to the graph $f(x) = x^2$ at the point (1, 1).

    (b) Suppose that the total cost in dollars incurred per weak by a company in manufacturing x refrigerators is given by the total cost function $c(x) = -0.2x^2 + 200x + 9000$, $0 \leq x \leq 400$.

    (i) What is the cost incurred in manufacturing the 201 st refrigerator ?

    (ii) Find the rate of change of c with respect to x when $x = 200$.

25. A man has 100 ft of fencing to enclose a rectangular garden. Find the dimensions of the garden of largest area he can have if he uses all of the fencing.

26. (a) Estimate $\int_0^t e^{-\sqrt{x}}\,dx$ using the property of definite integral.

    (b) Use the geometric interpretation of the integral to evaluate $\int_{-1}^{2} |x-1|\,dx$ by making a sketch of f.

27. Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis.

(2 × 10 = 20 marks)