C 40187 **(Pages : 5)** **Name.......................................**

Reg. No..................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION MARCH 2023

Mathematics

MAT 6B 11—NUMERICAL METHODS

(2017—2018 Admissions)

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. State Ramanujan's method to find a real root of the equation.

2. Form the forward difference table of the function $f(x)=\frac{1}{x}$ for the values $x=1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$.

3. State Newton's general interpolation formula with divided differences.

4. Evaluate $\Delta^2 (\cos 2x)$, interval of differencing being h.

5. State Gauss' backward central difference formula.

6. What do you mean by inverse interpolation ?

7. Given a set of n-values of (x, y), what is the formula for computing $\left[\frac{d^2y}{dx^2}\right]_{x_0}$.

8. Give Simpson's $1/3$-rule of integration.

9. What is Partial pivoting.

10. Find the eigenvalues of the matrix $\begin{bmatrix} 1 & 4 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$.

Turn over

11. In solving $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$ write down Taylor's series for $y(x_1)$.

12. State Predictor formula.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 *marks.*

13. Find by iteration method, a real root of the equation $x^3 = 1 - x^2$ on the interval $[0, 1]$ with an accuracy of 10^{-4}.

14. Prove that (i) $\mu = \cosh\left(\frac{hD}{2}\right)$; and (ii) $\delta = \Delta E^{-1/2}$.

15. Find the third divided difference with arguments 2, 4, 9, 10 of the function $f(x) = x^3 - 2x$.

16. Find the missing terms in the following table :

x	:	0	5	10	15	20	25	30
y	:	1	3	–	73	225	–	1153

17. Certain corresponding values of x and $\log_{10}x$ are (300, 2.4771), (304, 2.4829), (305,2.4843) and (307, 2.4871). Find $\log_{10} 301$.

18. Evaluate the integral $\int_{-2}^{2} \frac{x}{5 + 2x} dx$ using the trapezoidal rule with five ordinates.

19. Use Gauss elimination method to solve the system

$$2x + y + z = 10 ; 3x + 2y + 3z = 18 ; x + 4y + 9z = 16$$

20. Decompose the matrix $\begin{bmatrix} 4 & 3 & 2 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}$ in the form LU where L is a lower triangular matrix and U is unit upper triangular matrix.

21. If D stands for the differential operator $\frac{d}{dx}$, prove that $D = \frac{1}{h}\left[\Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \ldots\ldots\right]$.

22. Find the largest eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.

23. Given $\frac{dy}{dx} - 1 = xy$ and $y(0) = 1$, obtain the Taylor series for $y(x)$ and compute $y(0.1)$ correct to four decimal places.

24. The following table gives the population of a town during the last six censuses. Estimate, using Newton's interpolation formula, the increase in the population during the period 1946 to 1948 :

Year	:	1911	1921	1931	1941	1951	1961
Population (in thousands)	:	12	15	20	27	39	52

25. Determine the real root of the equation $x = e^{-x}$, using the secant method.

26. The distances (x cm) traversed by a particle at different times (t seconds) are given below :

t	:	0.0	0.1	0.2	0.3	0.4	0.5	0.6
x	:	3.01	3.16	3.29	3.36	3.40	3.38	3.32

Find the velocity of the particle at $t = 0.3$ seconds.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 *marks.*

27. Using Newton-Raphson's method, find a real root, correct to 3 decimal places, of the equation $\sin x = \frac{x}{2}$ given that the root lies between $\frac{\pi}{2}$ and π.

28. Find a root of the equation $4e^{-x} \sin x - 1 = 0$ by regula-falsi method given that the root lies between 0 an 0.5.

Turn over

29. Using the method of separation of symbols, show that

$$\Delta^n u_{x-n} = u_x - n u_{x-1} + \frac{n(n-1)}{2} u_{x-2} + \dots + (-1)^n u_{x-n}.$$

30. Values of x (in degrees) and $\sin x$ are given in the following table :

x	:	15	20	25	30	35	40
$\sin x$	:	0.2588190	0.3420201	0.4226183	0.5	0.5735766	0.6427876

Determine the value of sin 38.

31. Using Gauss's forward formula, find the value of $f(32)$ given that $f(25) = 0.2707$, $f(30) = 0.3027$, $f(35) = 0.3386$ and $f(40) = 0.3794$.

32. Given the table of values :

x	:	1.4	1.5	1.6	1.7
e^x	:	4.0552	4.4817	4.9530	5.4739

Use the method of successive approximations to find x and $e^x = 4.7115$.

33. From the following values of x and y, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 3$:

x	:	0	1	2	3	4	5	6
y	:	6.9897	7.4036	7.7815	8.1291	8.4510	8.7506	9.0309

34. Find the inverse of the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ using LU decomposition Method.

35. Given the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2 + 1}$ with the initial condition $y = 0$ when $x = 0$, use Picard's method to obtain y for $x = 0.25, 0.5$ and 1.0 correct to three decimal places.

($6 \times 7 = 42$ marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 marks.

36. a) From the following table, find the number of students who obtained marks between 60 and 70 :

Marks obtained	:	0-40	40-60	60-80	80-100	100-120
No. of Students	:	250	120	100	70	50

b) Find the Lagrange interpolating polynomial of degree 2 approximating the function $y = \ln x$ defined by the following table of values. Hence determine the value of ln 2.7.

x	:	2	2.5	3.0
$y = \ln x$	:	0.69315	0.91629	1.09861

37. Solve the system $8x - 3y + 2z = 20$; $6x + 3y + 12z = -35$; $4x + 11y - z = 33$ using both Jacobi and Gauss-Seidel method

38. a) Given the differential equation $\frac{dy}{dx} = x^2 + y$ with $y(0) = 1$, compute $y(0.02)$ using Euler's modified method.

b) Using Milne's method to obtain the value of $y(0.3)$ given that

$$\frac{dy}{dx} = x^2 + y^2 - 2,\ y(-0.1) = 1.0900,\ y(0) = 1.0000,\ y(0.1) = 0.8900 \text{ and } y(0.2) = 0.7605.$$

(2 × 13 = 26 marks)