

D 10666

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Name.....

Reg. No.....

FIFTH SEMESTER U.G. DEGREE EXAMINATION, NOVEMBER 2021

(CBCSS—UG)

Mathematics

MTS 5B 05—THEORY OF EQUATIONS AND ABSTRACT ALGEBRA

(2019 Admissions)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A

*Answer at least ten questions.**Each question carries 3 marks.**All questions can be attended.**Overall Ceiling 30.*

1. Show that $x^5 - 3x^4 + x^2 - 2x - 3$ is divisible by $x - 3$.
2. Factorize into linear factors the polynomial $x^4 - 1$.
3. Write a cubic equation with roots $1, 1 + i, 1 - i$.
4. State Identity theorem.
5. How many real roots has the equation $x^4 - 4ax + b = 0$.
6. Make addition and multiplication tables for \mathbb{Z}_2 .
7. Check whether the relation on \mathbb{R} defined by $a \sim b$ if $a - b \in \mathbb{Q}$ is an equivalence relation.
8. Consider the permutations $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$. Compute $\sigma\tau$ and $\tau\sigma$.
9. Let G be a group and $a, b \in G$. Show that $(ab)^{-1} = b^{-1}a^{-1}$.
10. Write a subgroup of $(\mathbb{Z}, +)$.

Turn over

11. Check whether $\mathbb{Z} \times \mathbb{Z}$ is cyclic.
12. Find order of the permutation $(1, 3)(2, 6)(1, 4, 5)$.
13. Let $\Phi : G_1 \rightarrow G_2$ be a group homomorphism. Show that $\Phi(e) = e'$ where e and e' are identity elements of G_1 and G_2 respectively.
14. Define a Ring.
15. Give example of an integral domain.

(10 × 3 = 30 marks)

Section B

*Answer at least **five** questions.*

Each question carries 6 marks.

All questions can be attended.

Overall Ceiling 30.

16. Solve $x^5 - 3x^4 + 4x^3 - 4x + 4$ having the root $1 + i$.
17. Solve the cubic equation $2x^3 - x^2 - 18x + 9 = 0$ whose roots are a, b, c with $a + b = 0$.
18. Find an upper limit of the positive roots of the equation $2x^5 - 7x^4 - 5x^3 + 6x^2 + 3x - 10 = 0$.
19. Prove that set of all even permutations of S_n is a subgroup of S_n .
20. Define $*$ on \mathbb{Z} by $a * b = a - b$. Check whether $(\mathbb{Z}, *)$ is a group.
21. Check whether \mathbb{Z}_n is cyclic.
22. Draw the subgroup diagram of \mathbb{Z}_{36} .
23. Let G_1 and G_2 be groups and let $\Phi : G_1 \rightarrow G_2$ be a function such that $\Phi(ab) = \Phi(a)\Phi(b)$ for all $a, b \in G$. Prove that Φ is 1-1 if and only if $\Phi(x) = e$ implies that $x = e$ for all $x \in G_1$.

(5 × 6 = 30 marks)

Section C

*Answer any **two** questions.
Each question carries 10 marks.*

24. Examine whether $x^4 - x^3 - x^2 + 19x - 42 = 0$ has integral roots or not.

25. Solve $x^3 - 6x - 6 = 0$ by Cardan's method.

26. Let G be a cyclic group. Show that :

(a) If G is infinite then $G \cong \mathbb{Z}$.

(b) If $|G| = n$, then $G \cong \mathbb{Z}_n$.

27. State and prove Lagrange's theorem.

(2 × 10 = 20 marks)