C 4402 **(Pages : 3)** **Name....................................**

Reg. No................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2021

Statistics

STA 2C 02—PROBABILITY THEORY

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical table are permitted

Section A (Short Answer Type Questions)

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Define (a) Random experiment ; (b) Event.
2. If the events A = { 1, 2, 3, 4, 5} and B = {4, 6, 7} are exhaustive events, identify the events : (i) $A \cap B^c$; (ii) $(A \cup B)^c$.
3. If P (A) = 0.4, P (B) = 0.5 and $P(A \cup B)$ = 0.7. Find $P(A/B^c)$
4. State multiplication theorem on probability for two events A and B. If P (A/B) = P (A) = P (B) = 0.4. Find $P(A \cup B)$.
5. Define probability density function and state any *two* of its properties.
6. Obtain the distribution function of X, with p.d.f. $f(x) = 3x^2$, for $0 < x < 1$.
7. Find the value of k, if $f(x) = \left(\frac{k}{2}\right)^x$, for x = 1, 2, 3... is the probability mass function of X.
8. If E (X) = 2, E (X^2) = 8, find V (3X – 2).
9. Obtain the mean and variance of a random variable X with m.g.f. $M_X(t) = (1-t)^{-1}$, $t < 1$.
10. Define characteristic function of a random variable and state its advantage over m.g.f.

Turn over

11. Find c, if $f(x, y) = c(x + 2y)$, for x = l,2 ; $-y = 0, 1$ is the joint p.m.f. of (X, Y).

12. Define independence of two random variables X and Y.

(8 × 3 = 24 marks)

Section B (Short Essay/Paragraph Type Questions)

Answer at least **five** *questions.*

Each question carries 5 *marks.*

All questions can be attended.

Overall Ceiling 25.

13. Mentioning the underlying assumptions clearly, state axiomatic definition of probability.

    Using this definition establish $0 \leq P(A) \leq 1$ for an event A.

14. A box contains 3 blue and 2 red balls. Another box contains 2 blue and 3 green balls. One of the identical boxes is selected and two balls were drawn without replacement. It is found that the two balls are blue. What is the probability that only green balls to remain in the selected box ?

15. The p.m.f. of X, $f(x) = \frac{2x^2 - 1}{k}$, for $x = 1, 2, 3, 4$ and $f(x) = 0$ *elsewhere* (i) Find k ; (ii) Write the distribution function $F(x)$.

16. Given the p.d.f. of X as $f(x) = 1$, for $0 < x < 1$. Find the p.d.f. of $Y = -2 \log_e X$.

17. In a game three balls are drawn from a box containing 5 white and 7 black balls. 10 points are given for each white ball drawn and 5 points are given for each black ball drawn. Calculated the expected points per game for a long run of the game.

18. For X with p.d.f. $f(x) = kx(2 - x)$, for $0 < x < 1$; $f(x) = 0$, elsewhere. Obtain (a) k ; (b) Mean and variance of X.

19. For two random variables X and Y, prove that (i) $V(X - Y) = V(X) + V(Y) - 2\,Cov(X, Y)$; (ii) $Cov(X - a, Y - b) = Cov(X, Y)$, where a and b are two constants.

(5 × 5 = 25 marks)

Section C (Essay Type Questions)

Answer any **one** *question.*
The question carries 11 *marks.*

20. (a) If A and B are two independent events prove that A^c and B^c are also independent.

    (b) Define the mutual independence of three events A, B and C. Also illustrate that the pairwise independence of A, B and C need not imply their mutual independence.

21. (a) Cauchy-Schwartz Inequality for two random variables X and Y.

    (b) Using this inequality prove $-1 \leq r_{XY} \leq +1$, where r_{XY} is the coefficient of correlation between X and Y.

(1 × 11 = 11 marks)