C 43189 **(Pages : 4)** **Name.....**

Reg. No.....

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2023

Mathematics

MTS 2B 02—CALCULUS OF SINGLE VARIABLE—1

(2019—2022 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 *marks.*
Maximum 25 *marks.*

1. Sketch the graph of the absolute value function $f(x) = |x|$.
2. By translating the graph of $y = x^2$, sketch the graphs of $y = x^2 + 2$.
3. Show that $\lim_{x \to 2} [x]$ does not exist.
4. State The Squeeze Theorem.
5. Show that zero is a critical number of each of the functions $f(x) = x^3$ and $g(x) = x^{1/3}$ but that neither function has a relative extremum at 0.
6. State Rolle's theorem and Mean Value Theorem.
7. Show that the function $f(x) = x^3 + x + 1$ has exactly one zero in the interval $[-2, 0]$.
8. Determine the intervals where the graph of $f(x) = x^{\frac{2}{3}}$ is concave upward and where it is concave downward.
9. Find (i) $\int 2x^3 \, dx$; and (ii) $\int (2x + 3\sin x) \, dx$.

Turn over

10. Verify the mean value theorem and find c for $f(x) = x(x-1)(x-2)$ for $a = 0, b = \frac{1}{2}$.

11. Evaluate $\frac{d}{dx}\int_0^x \frac{1}{2+t^4}\,dt$.

12. Using Riemann sum show that $\int_0^b x\,dx = \frac{b^2}{2}$.

13. Find the area of the region bounded by the graphs of $y = 2 - x^2$ and $y = -x$.

14. Use differentials to obtain an approximation of the arc length of the graph of $y = 2x^2 + x$ from $P(1, 3)$ to $Q(1.1, 3.52)$.

15. Find the area of the surface obtained by revolving the graph of $x = y^3$ on the interval $[0, 1]$ about the y-axis.

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Maximum 35 *marks.*

16. Show that the function $f(x) = |x|$ is differentiable everywhere except at 0.

17. Suppose that the total cost in dollars incurred per week by the Polaraire Corporation in manufacturing x refrigerators is given by the total cost function

$C(x) = -0.2x^2 + 200x + 9000 \quad 0 \le x \le 400.$

(i) What is the cost incurred in manufacturing the 201st refrigerator ?

(ii) Find the rate of change of C with respect to x when $x = 200$.

18. Find the points of inflection of $f(x) = (x-1)^{1/3}$.

19. Find $\lim_{x \to -\infty} \frac{x^2+1}{x-2}$.

20. If $y = \int_0^{x^3} \cos t^2 \, dt$, what is $\frac{dy}{dx}$?

21. Evaluate $\int_{-2}^{2} \frac{\sin x}{\sqrt{1+x^2}} \, dx$.

22. Find the volume of the solid obtained by revolving the region bounded by the graphs of $y = x^3$, $y = 8$ and $x = 0$ about the y-axis.

23. Find the center of mass of a system comprising three particles with masses 2, 3. and 5 slugs, located at the points $(-2, 2), (4, 6)$ and $(2, -3)$, respectively. (Assume that all distances are measured in feet.)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*
Maximum 20 marks.

24. (a) The total cost incurred in operating an oil tanker on an 800-mi run, traveling at an average speed of v mph. is estimated to be

$$C(v) = \frac{10,00,000}{v} + 200v^2$$

dollars. Find the approximate change in the total operating cost if the average speed is increased from 10 mph to 10.5 mph.

(b) Prove that $\lim_{\theta \to 0} \frac{\cos \theta - 1}{\theta} = 0$.

25. (a) Sketch the graph of the function $f(x) = \frac{1}{1 + \sin x}$.

(b) Find the vertical asymptotes of the graph of $f(x) = \tan x$.

Turn over

26. (a) Using Riemann sum evaluate $\int_{-1}^{3} \left(4 - x^2\right) dx$.

(b) Using the property of definite integral estimate the integral $\int_1^3 \sqrt{3 + x^2}\ dx$.

27. Find the area of the region bounded by the graphs of $x = y^2$ and $y = x - 2$.

(a) With respect to x ; and

(b) With respect to y.

(2 × 10 = 20 marks)