D 30180 **(Pages : 4)** **Name..**

Reg. No....................................

FIFTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION NOVEMBER 2022

Mathematics

MAT 5B 08—DIFFERENTIAL EQUATIONS

(2017—2018 Admissions)

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Fill in the blanks : The function $f(t)$ for which $\mathcal{L}\{f(t)\} = 1$ is ——————.

2. What can you say about the product of two odd functions ? Prove your assertion.

3. Write down the Euler formulas for computing the coefficients of a periodic function $f(t)$ of period 2L.

4. Write down the differential equation whose solution is $y = c_1 e^{2t} + c_2 e^{-2t}$.

5. Define the Wronskian of the functions $y_1(t)$ and $y_2(t)$.

6. Find inverse Laplace transform of $\dfrac{2(s-2)}{s+2}$.

7. What is the integrating factor of $x(x-2)\dfrac{dy}{dx} + y = \sin x$?

8. Find the order of the p.d.e. $\left(\dfrac{\partial u}{\partial x}\right)^7 - x^{-1/2}\, y^7 \dfrac{\partial^2 u}{\partial y^2} + x^2 y^3 \dfrac{\partial^2 u}{\partial x^2} = 7$.

9. Find the fundamental solutions of $y'' - 2y = 7t$.

Turn over

10. Write one dimensional wave equation ?

11. What is inverse Laplace transform of $t^{-1/2}$?

12. Solve the system : $\frac{dy}{dt} = x, \frac{dx}{dt} = 2$.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 *marks.*

13. Applying method of separation of variables, solve the BVP : $\frac{\partial y}{\partial t} = 2\frac{\partial y}{\partial x}$ with boundary conditions $y(x, 1) = y(x, 0) = 0$.

14. Write down the fundamental solutions of $y'' - 1 = y$ and find their Wronskians.

15. Convert $y'' + 2y' + 3y = 0$ into a system of first order equations.

16. Find the inverse Laplace transform of $\log((s-1)/(s-2))$.

17. Calculate the Laplace transform of Dirac's impulse function.

18. Solve the Cauchy's differential equation $t^2x'' - tx' - x = 0$.

19. State the existence and uniqueness theorem for first order differential equations with the assumptions involved therein.

20. Calculate a_n for $f(x) = 2x^2, x \in [-1, 1]$ in its Fourier series expansion.

21. Show that the Laplace transform is linear.

22. Solve : $\frac{dy}{dx} = \tan(x + y + 1)$.

23. Solve : $y' - y = 0$ using Laplace transform.

24. Solve the system : $\frac{dy}{dt} = 2x - y, \frac{dx}{dt} = x - 2y.$

25. State Abel's theorem.

26. Find the second order p.d.e. for which $y = f(x + ct) + g(x - ct)$ is a solution.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 marks.

27. Express the function $f(t) = \begin{cases} t \sin t, & \text{if } 0 \le t < \pi/2 \\ \cos t, & \text{if } \pi/2 \le t < \pi \\ 0, & \text{elsewhere} \end{cases}$ in terms of combination of unit step functions

and hence find its Laplace transform.

28. Evaluate the Laplace inverse transforms of $4 - \cot^{-1}(s/a)$ and $\frac{1}{\left(s^2 - 5s + 6\right)^2}$.

29. Find the solution by the checking the exactness of $\left(3y^2 - 2xy + 2\right)dx + \left(6xy - x^2 + y^2\right)dy = 0.$

30. State the conditions for the existence of Laplace transform of a function $f(t)$ and prove the same.

31. A ball with mass 0.15 kg is thrown upward with initial velocity 20 m/s from the roof of a building 30 m high. Neglect air resistance, (a) Find the maximum height above the ground that the ball reaches, (b) Assuming that the ball misses the building on the way down, find the time that it hits the ground.

32. Find the Fourier cosine series for the function $f(t) = \pi |t - \pi|, t \in [0, \pi]$.

33. Find the solution of the heat conduction problem :

$$25u_{xx} = u_t, 0 < x < 1, t > 0 \ ; \ u(0, t) = 0, u(1, t) = 0, \ t > 0 \ ; \ u(x, 0) = \sin(2\pi x) - \sin(5\pi x), 0 \le x \le 1.$$

Turn over

34. State and prove convolution theorem for Laplace transforms.

35. Evaluate (i) $\mathcal{L}^{-1}\left(\frac{1-e^{-2s}}{2s}\right)$; and (ii) $\mathcal{L}\left(t^2 \cos 2t - te^t \sin t\right)$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. (a) Apply method of variation of parameters to solve : $y'' + y = \sec t$.

(b) Solve $(3x + y + 1)\,dx + (x + 3y + 1)\,dy = 0$.

37. (a) Find the two half range Fourier series of the function: $f(t) = |\sin t|, t \in [0, \pi]$ and draw the graphs of the corresponding periodic extensions.

(b) Find an expression for $\mathcal{L}\left(\frac{f(t)}{t}\right)$ in terms of $\mathcal{L}(f(t))$ and prove the same.

38. (a) Derive the d'Alemberts solution of one dimensional wave equation.

(b) Find the solution of the p.d.e. $\frac{\partial^2 u}{\partial x^2} = 2$.

(2 × 13 = 26 marks)