D 50669 **(Pages : 2)** **Name.....**

Reg. No.....

FIFTH SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2023

Mathematics

MTS 5B 09—INTRODUCTION TO GEOMETRY AND THEORY OF EQUATIONS

(2020 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 *marks.*
Ceiling is 20.

1. Find equation of the normal to the parabola $y^2 = x$ at the point (1,1).
2. Find the foci of the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$.
3. What is the reflection property of the hyperbola ?
4. Find the matrix form of the conic $11x^2 + 4xy + 14y^2 - 4x - 28y - 16 = 0$.
5. Show that $x^3 + x^2 - 5x + 3$ is divisible by $x + 3$.
6. Write a cubic equation with the roots 1,2,3.
7. State the Identity Theorem.
8. Find the multiplicity of the root $x = 1$ of the polynomial $f(x) = x^n - nx + n - 1$.
9. Show that the polynomial $f(x) = x^{11} - 1$ has no roots in the interval $(-1,0)$.
10. Find Δ of the equation $x^3 - 10x - 12 = 0$.
11. Show that $\sqrt{2} - \sqrt{3}$ is a root of the equation $x^4 - 10x + 1 = 0$.
12. Find the cubic resolvent corresponding to the bi quadratic equation $x^4 + 4x - 1 = 0$.

(Ceiling 20)

Turn over

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Ceiling is 30.

13. Show that $t(x)=\begin{pmatrix}1 & 3\\ 1 & 2\end{pmatrix}x+\begin{pmatrix}4\\ -2\end{pmatrix}$ is an affine transformation and find the inverse.

14. (i) State the Fundamental theorem of Affine Geometry.

    (ii) Determine the affine transformation which maps the points (0, 0), (1, 0) and (0, 1) to the points (3, 2), (5, 8) and (7, 3), respectively.

15. Find the rational roots of the equation $6x^4-7x^3+8x^2-7x+2=0$.

16. Solve the equation $3x^3-16x^2+23x-6=0$ if the product of two roots is 1.

17. Factorize in to real linear and quadratic factors of the polynomial $f(x)=x^4+1$.

18. Show that the necessary and sufficient condition for an equation $x^3+px+1=0$ to have three real and distinct roots is $p^3<-27/4$.

19. How many real roots of the equation $f(x)=x^4-32x+1=0$.

(Ceiling 30)

Section C

Answer any **one** *question.*
The question carries 10 marks.

20. Prove that the conic with the equation $3x^2-10xy+3y^2+14x-2y+3=0$ is a hyperbola. Determine its centre, and its major and minor axis.

21. Solve the cubic equation $x^3+x^2-2=0$ by using Carden's formula.

(1 × 10 = 10 marks)