C 40974 (Pages : 4) Name.....

Reg. No.....

FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
APRIL 2023

Mathematics

MAT4B04—THEORY OF EQUATION, MATRICES AND VECTOR CALCULUS

(2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all **twelve** *questions.*
Each question carries 1 *mark.*

1. Write the number of roots of the equation $x^5 - 1 = 0$.
2. If α, β, γ are the roots of the equation $ax^3 + bx^2 + cx + d = 0$, write the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$.
3. Define a reciprocal equation.
4. Write the standard form of a biquadratic equation.
5. Find the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 3 & 2 & 1 \end{bmatrix}$.
6. If the rank of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is 2 and if $B = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$, find the rank of AB.
7. Find the number of solutions of the equation $4x - 7y = 0$.
8. If a system of two homogeneous linear equations in two unknowns has a non-zero solution, find rank of the co-efficient matrix of the system.
9. Find the characteristic roots of $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 4 \\ 0 & 0 & 5 \end{bmatrix}$.

Turn over

10. Write the parameter interval for the parametric equations of the line segment joining the points (1, 2, – 1) and (– 1, 0, 1).

11. Find the Cartesian equation for the surface $z = r^2$.

12. Find the curvature of the straight line $\mathbf{r}(t) = (i + j + \mathbf{k}) + t(2\mathbf{i} + \mathbf{j} + 3k)$.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. If α, β, γ are the roots of the equation $2x^3 + 7x^2 + 2x - 4 = 0$, find the value of $\alpha\beta + \beta\gamma + \gamma\alpha$

14. Find the number of roots of the equation $2x^3 - 7x^2 + 2x + 3 = 0$ between – 2 and 2.

15. Show that the equation $x^4 + 5x^2 + 10x - 2 = 0$ has exactly one negative root.

16. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \\ 2 & 4 & 8 \end{bmatrix}$.

17. Write the set of normal forms of nonzero matrices of order 4.

18. Find the values of a so that the system of equations $ax + 2y = 0, 2x + ay = 0$ has only the trivial solution.

19. Show that the characteristic roots of a diagonal matrix are the same as its diagonal elements.

20. If λ is an eigenvalue of a matrix A, prove that $\lambda + k$ is an eigenvalue of A + kI.

21. Find the plane through the points (0, 0, 1), (2, 0, 0) and (0, 3, 0).

22. The velocity of a particle moving in space is $\frac{d\mathbf{r}}{dt} = (\cos t)\,\mathbf{i} - (\sin t)\,\mathbf{j} + \mathbf{k}$. Find the position of the particle as a function of t if $\mathbf{r} = 2\mathbf{i} + \mathbf{k}$ when $t = 0$.

23. Find the unit tangent vector of the helix $\mathbf{r} = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + t\mathbf{k}$.

24. Evaluate $\int_0^1 \left(\mathbf{i} + t\mathbf{j} + t^2\mathbf{k}\right) dt$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. If α, β, γ are the roots of the equation $2x^3 + 3x^2 - x - 1 = 0$, find the equation whose roots are $2\alpha + 3, 2\beta + 3, 2\gamma + 3$.

26. If α, β, γ are the roots of the equation $x^4 + 4x^3 + 6x^2 + 8x + 1 = 0$, find the value of $\sum (\alpha - \beta)^2$.

27. By cardan's method, find the real root of equation $x^3 - 18x = 35$.

28. Reducing to the nromal form, find the rank of $\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$.

29. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ by elementary row transformations.

30. Test for consistency and solve the system of equations :

$$\begin{aligned} x + 2y - z &= 3 \\ 3x - y + 2z &= 1 \\ 2x - 2y + 3z &= 2 \\ x - y + z &= -1. \end{aligned}$$

31. If A is a non-singular matrix, prove that the eigenvalues of A^{-1} are the reciprocals of the eigenvalues of A.

32. Find the point in which the line $x = 1 - t, y = 3t, z = 1 + t$ meets the plane $2x - y + 3z = 6$.

33. Find the curvature of a circle of radius a.

(6 × 5 = 30 marks)

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve the equation $3x^5 - 10x^4 - 3x^3 - 3x^2 — 10x + 3 = 0$.

Turn over

35. Find the characteristic roots and the corresponding characteristic vectors for the matrix

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}.$$

36. Find the binormal vector and torsion for the space curve $\mathbf{r}(t) = (4 \sin t)\,\mathbf{i} + (4 \cos t)\,\mathbf{j} + 3t\mathbf{k}$.

(2 × 10 = 20 marks)