**C 4185** **(Pages : 3)** **Name..................................**

**Reg. No..................................**

# SECOND SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, APRIL 2021

Statistics

STS 2C 02—PROBABILITY DISTRIBUTIONS

Time : Three Hours Maximum : 80 Marks

## Section A

*Answer* **all** *questions in* **one** *word.*
*Each question carries* 1 *mark.*

Name the following :

1. The coefficient of $\frac{(it)^r}{r!}$ in the expansion of characteristic function.
2. The discrete distribution having memoryless property.
3. The distribution of $\frac{X_1}{X_2}$ where $X_1$ and $X_2$ are independent gamma variables with parameters $n_1$ and $n_2$ respectively.

Fill up the blanks :

4. If X and Y are two independent variables, the conditional distribution of X given $Y = y, f(x \mid y) =$ ——————.
5. If $X \sim B(n, p)$, the distribution of $y = n - X$ is ——————.
6. If $X \sim N(\mu, \sigma^2)$, the points of inflexion of normal curve are ——————.
7. The variance of the rectangular distribution $f(x) = \frac{1}{b-a}; a \le x \le b$ is equal to ——————.

Write true or false :

8. If X, Y and Z are three random variables, then $\text{cov}(X+Y, Z) = \text{cov}(X, Z) + \text{cov}(Y, Z)$.
9. For a geometric distribution mean is always less than the variance.
10. The existence of variances of the random variables is not necessary for applying weak law of large numbers.

(10 × 1 = 10 marks)

**Turn over**

## Section B

*Answer* **all** *questions in one sentence each.*
*Each question carries* 2 *marks.*

11. Define mathematical expectation of a random variable.
12. What are the properties of moment generating function ?
13. Define conditional variance.
14. Define joint raw moments for the bivariate distribution.
15. Define geometric distribution.
16. If a random variable $X \sim N(40, 5^2)$, find $P(32 < X \leq 50)$.
17. Define convergence in probability.

(7 × 2 = 14 marks)

## Section C

*Answer any* **three** *questions.*
*Each question carries* 4 *marks.*

18. State and prove the addition theorem of expectation.
19. What are the physical conditions for which binomial distribution is used ?
20. Show that in a Poisson distribution with unit mean, mean deviation about mean is $\frac{2}{e}$ times the standard deviation.
21. Define beta distributions of Type I and Type II. Give the relation between them.
22. State and prove Bernoulli's weak law of large numbers.

(3 × 4 = 12 marks)

## Section D

*Answer any* **four** *questions.*
*Each question carries* 6 *marks.*

23. What is the expectation of the number of failures before the first success in an infinite series of independent trials with constant probability p of success in each trial ?
24. Two random variables X and Y have the following joint probability density function :

$$f(x,y) = \begin{cases} 2 - x - y, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the covariance between X and Y.

25. Find the m.g.f. of the random variables whose moments are (i) $\mu_r' = (r+1)!2^r$ and (ii) $\mu_r' = r!$

26. A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as Poisson variate with mean 1.5. Calculate the proportion of days on which (i) neither car is used and (ii) some demand is refused.

27. In a distribution exactly normal, 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of the distribution ?

28. Let $X_i$ assume the values $+ i$ and $- i$ with equal probabilities, show that law of large numbers cannot be applied to the independent variables $X_1, X_2, \ldots$

(4 × 6 = 24 marks)

**Section E**

*Answer any* **two** *questions.*
*Each question carries* 10 *marks.*

29. Prove that characteristic function is uniformly continuous.

30. Derive Poisson distribution as a limiting case of binomial distribution.

31. Explain the properties of normal distribution.

32. State and prove the Chebychev's inequality.

(2 × 10 = 20 marks)