C 22091 **(Pages : 3)** **Name.....**

Reg. No.....

SECOND SEMESTER (CBCSS-UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MTS 2B 02—CALCULUS OF SINGLE VARIABLE—I

(2021 Admissions)

Time : Two Hours and a Half Maximum Marks : 80

Section A

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Let $f(x) = \sin x$ and $g(x) = 1 - 2x$. Find the functions *gof* and *fog*. What are their domains ?
2. Find $\lim_{x \to -3} \frac{x^2 + 2x - 3}{x^2 + 4x + 3}$.
3. Let $H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0. \end{cases}$ Determine whether H is continuous from the right at 0 and/or from the left at 0.
4. Find $\lim_{x \to 0} \frac{\sin 2x}{3x}$.
5. Find the instantaneous rate of change of $f(x) = \frac{2}{x} + x$ at $x = 1$.
6. Find the derivative of $f(x) = 3\sqrt{x} + 2e^x$.
7. Find the critical points of $f(x) = x - 3x^{1/3}$.
8. State Mean value theorem.
9. Find $\lim_{x \to -\infty} \frac{x^2 + 1}{x - 2}$.
10. Find the horizontal and vertical asymptotes of $f(x) = \frac{1}{x + 2}$.
11. Find $\int \frac{2x^2 - 1}{x^2}\, dx$.

Turn over

12. Find $\int \frac{e^{2/x}}{x^2}\,dx$.

13. Evaluate $\int_{-1}^{2} |x|\,dx$.

14. Find the area of the region between the graphs of $y = e^x$ and $y = x$ and the vertical lines $x = 0$ and $x = 1$.

15. Find the work done by the force $F(x) = 3x^2 + x$ in moving a particle along the x-axis from $x = 2$ to $x = 4$.

(10 × 3 = 30 marks)

Section B

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Find $\lim_{x\to 0} x^2 \sin\frac{1}{x}$.

17. Let $f(x) = 2x^3 + x$ (a) Find $f'(x)$. (b) What is the slope of the tangent line to the graph of f at (2, 18) ; (c) How fast is f changing when $x = 2$.

18. Find the relative extrema of $f(x) = x^3 - 3x^2 - 24x + 32$ using second derivative test.

19. Let $f(x) = x^3 - x$ for x in [– 1, 1] :
    (a) Show that f satisfies the hypothesis of Rolle's theorem on [– 1, 1].
    (b) Find the numbers c in (– 1, 1) such that $f'(c) = 0$ by Roll's theorem.

20. (a) In a test run of a maglev along a straight elevated monorail track, data obtained from reading its speedometer indicated that the velocity of the maglev at time t can be described by the velocity function $v(t) = 8t, 0 \le t \le 30$. Find the position of the maglev. Assume that the maglev is initially located at the origin of a co-ordinate line.

    (b) Find $\int \frac{dx}{1-\sin x}$.

21. (a) State fundamental theorem of Calculus.

    (b) Find $\frac{d}{dx}\left[\int_1^x t^3 dt\right]$ by using the above theorem and by performing the integration and differentiation.

22. Let R be the region bounded by the graphs of $x = -y^2 + 6y$ and $x = 0$. Find the volume of the solid obtained by revolving R about the x-axis.

23. Find the area of the surface obtained by revolving the graph of $x = y^3$ on the interval [0, 1] about y-axis.

(5 × 6 = 30 marks)

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) By translating the graph of $y = x^2$, sketch the graphs of $y = x^2 + 2$ and $y = (x - 2)^2$.

(b) Let $f(x) = \begin{cases} -x^5 + x^3 + x + 1 & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ x^2 + \sqrt{x+1} & \text{if } x > 0 \end{cases}$

Find $\lim_{x \to 0^+} f(x)$ and $\lim_{x \to 0^-} f(x)$. Does $\lim_{x \to 0} f(x)$ exist. Justify your answer.

25. Sketch the graph of the function $f(x) = 2x^3 - 3x^2 - 12x + 12$.

26. Using the definition of the definite integral evaluate $\int_{-1}^{3} (4 - x^2)\, dx$.

27. (a) Find the area of the region enclosed by the graphs of $y = \frac{x^2}{4}$ and $y = \frac{8}{x^2 + 4}$.

(b) Find the volume of a right pyramid with a square base of side b and height h.

(2 × 10 = 20 marks)