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Reg. No..................................

FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2022

Mathematics

MTS 5B 05—THEORY OF EQUATIONS AND ABSTRACT ALGEBRA

(2019 Admissions only)

Time : Two Hours and a Half — Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Ceiling is 25.

1. Find the quotient and remainder when $-x^4+7x^3-4x^2$ is divided by $x-3$.

2. Write a cubic equation with roots $1, 1+i, 1-i$.

3. Factorise into real linear and quadratic factors : $x^4+x^3+x^2+x+1$.

4. State Rolles Theorem.

5. Verify whether the equation $x^3-3x^2-4x+13=0$ has roots in the interval $(-3,-2)$.

6. Define Euler's function. Find the value of $\varphi(36)$.

7. Find the multiplicative inverse of (14) in $\mathbb{Z}_{15}$.

8. Prove that if p is a prime $(p-1)!\equiv -1 \pmod{p}$.

9. Check whether the relation $\sim$ on R defined by $a \sim b$ if $|a-b| \le 1$ is an equivalence relation.

10. Let $\sigma, \tau \in S_7$ be given by $\sigma=(1\,3\,5\,6)$ and $\tau=(1\,2)(3\,5\,4\,7)$. Find $\sigma\tau\sigma^{-1}$.

11. Find the order of $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 4 & 9 & 6 & 5 & 2 & 3 & 1 & 7 \end{pmatrix}$.

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12. Let G be a group. Prove that if $x^2 = e$ for all $x \in G$ then G is abelian.

13. Write the subgroups of S_3.

14. Let G be a group with $a, b \in G$ then show that $O\left(aba^{-1}\right) = O(b)$.

15. Let $G = \mathbb{Z}_{12}$ and $H = 4\,\mathbb{Z}_{12}$. Find all cosets of H.

Section B

Answer any number of questions.
Each question carries 5 marks.
Ceiling is 35.

16. Solve the biquadratic equation $x^4 - 7x^3 + 18x^2 - 22x + 12 = 0$ whose roots are a, b, c and d and if $ab = 6$.

17. Find the limits of roots of the equation $2x^5 - 8x^4 - 11x^3 + 5x^2 + 2x - 11 = 0$.

18. Separate the roots of the equation $3x^4 - 2x^3 - 6x^2 + 6x - 2 = 0$.

19. Let G be a group and $a, b \in G$. Prove that $(ab)^n = a^n b^n$ for all $n \in \mathbb{Z}$ iff $ab = ba$.

20. Find all cyclic subgroups of $\mathbb{Z}_8$.

21. Check whether the set $\left\{a + b\sqrt[3]{2} \mid a, b \in \mathbb{Q}\right\}$ are subring of the field of real numbers.

22. Show that the cyclic group $< i >$ is isomorphic to $\mathbb{Z}_4$.

23. Let $\varphi : G \to H$ is an isomorphism of groups. Then show that :

    a) If G is abelian then so is H ; and

    b) If G is cyclic then so is H.

Section C

Answer any **two** *questions.*

Each question carries 10 *marks.*

Maximum marks 20.

24. Solve the cubic equation : $2x^3 + 3x^2 + 3x + 1 = 0$.

25. Let G be a group with normal subgroups H and K such that $HK = G$ and $H \cap K = \{e\}$ then show that $G \cong H \times K$.

26. Let G be a cyclic group. Then show that :

    a) If G is infinite then $G \cong \mathbb{Z}$.

    b) If G is finite with $|G| = n$, then $G \cong \mathbb{Z}_n$.

27. State and prove Fundamental Homomorphism theorem.

(2 × 10 = 20 marks)