D 110212 (Pages : 3) Name.....

Reg. No.....

FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2024

Mathematics

MTS 5B 09—INTRODUCTION TO GEOMETRY AND THEORY OF EQUATIONS

(2020 Admission onwards)

Time : Two Hours Maximum : 60 Marks

Section A

Not more than 20 *marks can be earned from this unit.*
Each question carries 2 *marks.*

1. State Reflection Property of the Parabola.
2. Determine the equation of the tangent to the ellipse with parametric equations

   $x = 3\cos t, y = \sin t$

   at the point with parameter value $t = \pi/4$.
3. Prove that Euclidean-congruence is a symmetric relation.
4. Give the inverse of the affine transformation $t(\boldsymbol{x}) = \mathbf{A}\boldsymbol{x} + \boldsymbol{b}$.
5. Find the quotient and remainder obtained when $f(x) = 2x^7 - 3x^6 + x^5 - 3x^4 + 5x^3 - 4x^2 + 2x - 1$ is divided by $g(x) = 2x^3 - 3x^2 + x - 1$.
6. Calculate the values of the polynomial $4x^3 - 7x^2 + 5x + 3$ and their derivatives for the value of $x = -2$.
7. State the Fundamental theorem of Algebra.
8. Verify that i is a zero of $f(x) = x^3 + 2x - i$
9. How many real roots has the equation $x^4 - 4ax + b = 0$?

Turn over

10. Verify that the equation $x^3 - 7x + 7 = 0$ has roots in the interval $\left(\frac{3}{2}, 2\right)$.

11. State True/False : Let α, β, γ are the roots of the equation $f(x) = 0$, then $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}, \cdots$ are the roots of the equation $f\left(\frac{1}{x}\right) = 0$.

12. State True/False : If the equation contains only even powers of x and the co-efficients are all of the same sign, the equation has no real root.

Section B

Not more than 30 *marks can be earned from this unit.*
Each question carries 5 *marks.*

13. Prove that 2×2 matrix **P** represents a rotation of $\mathbb{R}^2$ about the origin if and only if it satisfies the following two conditions :

    (a) **P** is orthogonal ;

    (b) det **P** = 1.

14. Determine the affine transformation which maps the points (2, 3), (1, 6) and (3, – 1) to the points (1, – 2), (2, 1) and (– 3, 5), respectively.

15. Show that the roots of the equation

    $$x^3 + px^2 + qx + r = 0$$

    are in arithmetic progression if $2p^3 - 9pq + 27r = 0$.

16. If α, β, γ are roots of $x^3 + px^2 + qx + r = 0$, find the values of $\sum \frac{1}{\beta\gamma}$ in terms of co-efficients of the equation.

17. Find an upper limit of the positive roots of the equation

    $$x^5 - 7x^4 - 100x^3 - 1000x^2 + 10x - 50 = 0.$$

18. Find the rational roots of the equation $6x^4 - 7x^3 + 8x^2 - 7x + 2 = 0$.

19. Using Descartes' Rule of signs, show that the equation :

$$x^6 - x^3 + 2x^2 - 3x - 1 = 0$$

has four imaginary roots.

Section C

Answer any **one** *question.*
Each question carries 10 *marks.*

20. (a) Prove that an affine transformation maps parallel straight lines to parallel straight lines.

(b) If α, β and γ are the roots of the equation $x^3 + ax^2 + bx + c = 0$, form, the equation whose roots are $\alpha\beta$, $\beta\gamma$ and $\gamma\alpha$.

21. (a) Solve the biquadratic equation $x^4 - 3x^2 + 6x - 2 = 0$.

(b) Solve $x^3 - 6x^2 + 3x - 2 = 0$ by Cardano's method.

(1 × 10 = 10 marks)