

C 43191

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Name.....

Reg. No.....

**SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2023**

Mathematics

MTS 2C 02—MATHEMATICS—2

(2020—2022 Admissions)

Time : Two Hours

Maximum : 60 Marks

Section A*Answer any number of questions.**Each question carries 2 marks.**Maximum marks that can be earned from this Section is 20.*

1. Sketch the set of points whose polar co-ordinates (r, θ) satisfy the conditions $0 < r < 4$ and $-\pi/2 < \theta < \pi/2$.
2. Let $f(x) = x^2 + 2x + 3$. Restrict f to a suitable interval so that it has an inverse. Find the inverse $f^{-1}(x)$ of the given function.
3. Find the slope of the line tangent to the graph of $r = \cos(3\theta)$ at $\theta = \pi/3$.
4. Prove $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$.
5. Show that determinant of a square matrix A is the product of its eigenvalues
6. Find $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ using the notion of improper integrals.
7. Evaluate $\lim_{n \rightarrow \infty} \frac{n^2 + 1}{3n^2 + n}$.
8. Define rank of a matrix.

Turn over

9. Find A^2 using Cayley-Hamilton theorem, if $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.
10. When will you say that a series $\sum_{n=1}^{\infty} a_n$ converges to the sum s .
11. Use Doolittle's method to find an LU-factorization of $B = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$.
12. Selecting a proper test of convergence, decide whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2 - \ln n}$ converges or diverges.

Section B

Answer any number questions from this section.

Each question carries 5 marks.

Maximum that can be earned from this Section is 30.

13. Find the maxima and minima of $f(\theta) = 1 + 2 \cos(\theta)$. Sketch the graph of $r = 1 + 2 \cos \theta$ in the xy -plane.
14. Find :
- (a) $\frac{d}{dx}(\sinh^{-1}(3x))$; and
- (b) $\int \sinh^{-1}(3x) dx$.
15. Prove that the length of the parabola $y = x^2$ from $x = 0$ to $x = 1$ is $\frac{1}{2} \left[\sqrt{5} + \frac{1}{2} \ln(2 + \sqrt{5}) \right]$.
16. A bouncing ball loses half of its energy on each bounce. The height reached on each bounce is proportional to the energy. Suppose that the ball is dropped vertically from a height of one meter. How far does it travel ?

17. Find the Taylor's series expansion for $\ln x$ around $x = 1$.

18. Find the rank of the matrix $A + 3I$ where $A = \begin{bmatrix} 0 & 1 & 1 & 7 \\ 1 & 0 & 1 & -3 \\ 4 & 1 & 0 & 3 \\ 4 & 1 & 0 & 3 \end{bmatrix}$.

19. The set $B = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$, where $\mathbf{u}_1 = \langle 1, 1, 1 \rangle$, $\mathbf{u}_2 = \langle 1, 2, 2 \rangle$, $\mathbf{u}_3 = \langle 1, 1, 0 \rangle$ is a basis for $\mathbb{R}^3$. Transform B into an orthonormal basis.

Section C

Answer **one** question from this section.

The question carries 10 marks.

Maximum that can be earned from this Section is 10.

20. (a) Find the surface area of a sphere of radius r using the method of integration.

(b) Use Simpson's rule of integration to evaluate $\int_0^1 \frac{dx}{x^2 + 1}$ and hence find an approximate value for π .

21. (a) If consistent solve the system using Gauss-Jordan elimination :

$$2x_1 + x_2 + x_3 = 3$$

$$3x_1 + x_2 + x_3 + x_4 = 4$$

$$x_1 + 2x_2 + 2x_3 + 3x_4 = 3$$

$$4x_1 + 5x_2 - 2x_3 + x_4 = 16.$$

(b) Diagonalize the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

(1 × 10 = 10 marks)