C 40186 **(Pages : 4)** Name.......................................

Reg. No..................................

SIXTH SEMESTER (CUCBCSS–UG) DEGREE EXAMINATION, MARCH 2023

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

(2017–2018 Admissions)

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *the twelve questions.*
Each question carries 1 *mark.*

1. Solution of $e^z = 3$ is ————.
2. The principal value of logarithm of $-i$ is ————.
3. An analytic function with constant argument is ————.
4. The polar form of the Cauchy–Riemann equation for $f(x) = u(x,y) + i\nu(x,y)$ is ————.
5. The value of $\int_{|z|=2} \bar{z}\, dz =$ ————.
6. State Cauchy–Goursat's theorem.
7. $\int_{|z|=1} \frac{cosz}{z}\, dz =$ ————.
8. State Liouvelle's theorem.
9. The Radius of Convergence of the power series $\sum \frac{(i)^\eta}{3^\eta} n^2 z^\eta$ is ————.
10. The complex number $z = z_0$ is an essential singular of $f(z)$ if ————.
11. Define Residue of a complex function.
12. $\int_{|z|=z} e^{1/z^2} dz =$ ————.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any **ten** *out of* 14 *questions.*
Each question carries 4 *marks.*

13. Show that $f(z) = e^{\bar{z}}$ is nowhere analytic.

14. Prove that an analytic function with constant magnitude reduces to a constant.

15. If $f(z) = u + iv$ is analytic, then prove that $\frac{\partial f}{\partial x} = -i\frac{\partial f}{\partial y}$.

16. If $v(x, y)$ is the harmonic conjugate of $u(x, y)$ then prove that $u(x, y)$ is the harmonic conjugate of $-v(x, y)$.

17. Find an analytic function whose real part is $u(x, y) = e^x(x\cos y - y\sin y)$.

18. Find the principal value of $(1+i)^i$.

19. Evaluate $\int_C \frac{z^2 - 1}{z^2 + 1} dz$, where $C = |z + i| = 1$ taken in the positive sense.

20. (a) State the fundamental theorem of complex integration.

    (b) Evaluate $\int_{-i\pi}^{i\pi} \cos z dz$.

21. Evaluate the integral $\int_{|z|=1} \frac{e^{2z} dz}{z^4}$.

22. If $f(z)$ is continuous through out a Domain D and if $\int_C f(z)\, dz = 0$ for every simple closed curve C in D, then prove that $f(z)$ is analytic in D.

23. (a) State Laurent's theorem.

    (b) Expand $f(z) = e^{1/z}$ as a Laurent's series.

24. (a) What do you mean by singular points of a complex function ?

(b) Determine the singular points if any for the function $f(z) = \dfrac{z^7}{\left(z^4 - 1\right)^2}$.

25. (a) State Cauchy's residue theorem.

(b) Evaluate $\displaystyle\int_{|z|=1} \frac{\sin \pi z \, dz}{z^6}$.

26. Expand $f(z) = \dfrac{z-1}{z+1}$ as a Taylor series about $z = 0$.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of* 9 *questions.*
Each question carries 7 *marks.*

27. If $f(z) = u + iv$ is analytic, then prove that $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4\left|f'(z)\right|^2$.

28. (a) What do you mean by harmonic conjugate ?

(b) Find the harmonic conjugate of $u(x, y) = e^{-x}(x \sin y - y \cos y)$.

29. Expand $f(z) = \dfrac{1}{(z+1)(z+3)}, 1 < |z| < 3$ as a Laurent series.

30. State and prove Cauchy's integral formula.

31. When do we say the singularity of a complex function isolated ? What are the different types of isolated singularities ? Give examples for each.

32. Evaluate (a) $\displaystyle\int_{1+i}^{2+3i} \left(12z^2 + 4\,iz\right) dz.$ (b) $\displaystyle\int_{|z|=1/2} \frac{(2z-1)\,dz}{z^2 - z}$.

33. If $f(z)$ has a pole of order on at $z = z_0$, prove that $\dfrac{1}{f(z)}$ has a zero of order m at $z = z_0$.

Turn over

34. Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$ by the method of Residues.

35. Evaluate $\int_0^{\infty} \frac{dz}{\left(1+x^2\right)^2}$ by using Residues.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of* 3 *questions.*
Each question carries 13 *marks.*

36. (a) State and prove the fundamental theorem of Algebra.

(b) Prove that $\int_0^{2\pi} \frac{d\theta}{1+a\cos\theta} = \frac{2\pi}{\sqrt{1-a^2}}, -1 < a < 1.$

37. (a) Prove that the zeros of an analytic function are isolated.

(b) Find $\operatorname*{Res}_{z=i} f(z)$, if $f(z) = \frac{(\ln z)^3}{z^2+1}$.

38. (a) Evaluate $\int_C \frac{dz}{\left(z^3-1\right)^2}$, where $c = |z-1| = 1$.

(b) Discuss the nature of singularities of $f(z) = \sin\left(\frac{1}{z}\right)$.

(c) Does the function $f(z) = \sin z$ bounded ? Justify.

(2 × 13 = 26 marks)