D 12646 **(Pages : 3)** **Name.....**

Reg. No.....

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2021

Mathematics

MTS 1C 01—MATHEMATICS—I

(2021 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Calculate the slope of the tangent line to the graph of $f(x) = x^2 + 1$ when $x = -1$.

2. Find $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - x}$.

3. Find the derivative of $y = \sqrt{x}$ for $x > 0$.

4. Find $\frac{d}{dx}\left[\cos\left(\sqrt{1 + \cos x}\right)\right]$.

5. Find the linearization of $f(x) = \cos x$ at $x = \pi/2$.

6. Show that there is a number c such that $c^3 - c^2 = 10$.

7. Find $\lim_{t \to 0} \cos\left(\frac{x}{\sqrt{19 - 3\sec 2t}}\right)$.

8. Suppose that f is differentiable on the whole real line and that $f'(x)$ is constant. Prove that f is linear.

Turn over

9. Find the critical points of $f(x) = 3x^4 - 8x^3 + 6x^2 - 1$.

10. Find the inflection points of $f(x) = x^2 + (1/x)$.

11. Using limits of Riemann sums, establish the equation $\int_a^b c\,dx = c(b-a)$, where c is a constant.

12. Find $\int_0^2 \left(\frac{t^2}{4} - 7t + 5\right) dt$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Find $\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$.

14. Show that the line $y = mx + b$ is its own tangent at any point $(x, mx + b)$ on the line.

15. Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 1 ft/s. How fast is the area of the spill increasing when the radius of the spill is 20 ft ?

16. Use implicit differentiation to find d^2y/dx^2 if $5x^3 - 7y^2 = 10$.

17. Find the maximum and minimum points and values for the function $f(x) = \left(x^2 - 8x + 12\right)^4$ on the interval $[-10, 10]$.

18. Use l'Hôpital's Rule to find $\lim_{x \to 0} \frac{\sin x - x}{x^3}$.

19. Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x, -1 \leq x \leq 2$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. (a) Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

(b) Find $\frac{dy}{dx}$ if $y = \int_1^{x^2} \cos t \, dt$.

21. (a) Find the absolute extrema of $h(x) = x^{2/3}$ on $[-2, 3]$.

(b) Find the volume of the solid generated by the revolution about the x-axis of the loop of the curve $y^2 = x^2 \frac{3a - x}{a + x}$.

(c) Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$.

(1 × 11 = 11 marks)