C 43209 (Pages : 2) Name.....

Reg. No.....

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2023

Statistics

STA 2C 02—PROBABILITY THEORY

(2019—2022 Admissions)

Time : Two Hours Maximum : 60 Marks

Use of Calculator and Statistical tables are permitted.

Part A (Short Answer Type Questions)

Each question carries 2 *marks.*
Maximum marks that can be scored from this part is 20.

1. Define field and sigma field.
2. State addition theorem of probability for two events.
3. Define the terms : (a) Mutually exclusive events ; and (b) Independent events.
4. If P (A) = 0.3, P (B) = 0.2 and $P(A \cap B) = 0.1$, find the probability that exactly one of the events will happen.
5. Define random variable.
6. Define pdf and give the properties of p.d.f.
7. A die is tossed until an odd number appears. Obtain the probability distribution of the number of tosses.
8. Three unbiased coins are tossed. Find the expectation of the number of heads.
9. What are the limitations of moment generating function.
10. What is meant by skewness ?
11. The joint p.d.f. of a two dimensional random variable(X, Y) is given by $f(x, y) = 2$, $0 < x < 1$, $0 < y < x$ and 0 elsewhere. Find the marginal density functions of X and Y.
12. How do you interpret correlation co-efficient ?

Turn over

Part B (Short Essay/Paragraph Type Questions)

Each question carries 5 *marks.*

Maximum marks that can be scored from this part is 30.

13. Define conditional probability of two events. If A and B are independent, show that (a) A and B^c are independent ; and (b) A^c and B^c are independent.

14. Define independence of two random variables. Give one example to show that pairwise independence does not imply mutual independence.

15. A continuous random variable X has the p.d.f. $f(x) = 3x^2, 0 \leq x \leq 1$. Find two numbers a and b such that : (i) $P(x \leq a) = P(x \geq a)$; and (ii) $P(x \geq b) = 0.05$.

16. $f(x) = \frac{x}{15}$, when $x = 1, 2, 3, 4, 5$ and 0 elsewhere is the density function of the random variable X. Find its distribution function.

17. Two unbiased dice are thrown. Find the expectation of the sum of the number of points on them.

18. Find the m.g.f of $f(x) = \frac{1}{2} e^{-|x|}, \ -\infty < x < \infty$.

19. Joint distribution of X and Y is given by : $f(x, y) = 4xy\, e^{-(x^2 + y^2)}, x \geq 0, y \geq 0$. Test whether X and Y are independent. Also find the conditional density of X given Y = y.

Part C (Essay Type Questions)

Answer any **one** *question.*

The question carries 10 *marks.*

Maximum marks that can be scored from this part is 10.

20. State Bayes' theorem. There are two identical boxes containing respectively 4 white and 3 red balls; 3 white and 7 red balls. A box is chosen at random and a ball is drawn from it. Find the probability that it is from the first box ?

21. Let $f(x, y) = 8xy$, $0 < x < y < 1$; $f(x, y) = 0$ elsewhere. Find (i) E (Y/X = x) ; (ii) E(XY/X = x) ; and (iii) Var(Y/X = x).