D 12510 **(Pages : 4)** **Name.......................................**

Reg. No..................................

FIRST SEMESTER (CUCBCSS-UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MAT 1C 01—MATHEMATICS

(2016—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type Questions)

Answer **all** *questions (1-12).*
Each question carries 1 mark.

1. $\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\frac{\pi}{2} - x} = ...$

2. State sandwich theorem for limits.

3. What is a jump discontinuity ?

4. State Max-Min theorem for continuous functions.

5. Define point of inflection of a function $y = f(x)$.

6. What are the asymptotes of $y = \tan x$.

7. If $y = x^4 - 3 \cos x + e^x$, $dy = ...$

8. Find the critical points of $f(x) = x^3 + 12x + 5$, in $[-3, 3]$.

9. When we say that a function $y = f(x)$ is concave up in $[a, b]$?

10. If f and g are two monic polynomials (leading coefficient is 1) of same degree, what is $\lim_{x \to \infty} \frac{f(x)}{g(x)}$?

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11. What is Riemann sum for a function f on the interval $[a, b]$.

12. If $f(x) > 0$, what is the area of the region bounded by the the graph of f, the x-axis and the ordinates $x = a$ and $x = b$.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions (13-24).*
Each question carries 2 mark.

13. Using formal definition of limit, show that $\lim_{x \to 1} (5x - 3) = 2$.

14. Using intermediate value theorem, show that there is a real number which is exactly one less than its cube.

15. Find left and right limits of the function f at $x = 2$, where $f(x) = \begin{cases} 3 - x & x \le 2 \\ \frac{x}{2} + 1, & x > 2. \end{cases}$

16. Let $f(x) = -x^3 + 12x + 5,\ x \in [-3, 3]$. Where does the function f assume extreme values and what are these values ?

17. Define removable discontinuity and give an example.

18. Verify Rolle's theorem for the function $f(x) = (x - 2)(x - 3)$ on the interval [2, 3].

19. Find the horizontal/vertical asymptotes of the graph of $f(x) = \frac{x^3 - 1}{x^2 - 1}$.

20. Find the average of $y = 2x - x^2$ in [0, 3].

21. Find the linearization of $f(x) = 2 - \int_2^{x+1} \frac{9}{1+t}\, dt$.

22. Find dy/dx if $y = \int_x^1 \sqrt{1 + t^2}\, dt$. Explain main steps in your calculation.

23. Find the area between $y = \sin x$, $x = -\pi/2, x = \pi/2$ and the x-axis.

24. Write down the main steps to find the volumes of solids by the method of slicing.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions (25-33).*
Each question carries 5 marks.

25. Define continuity and different types of discontinuity of a function $f(x)$ at a point a.

26. State Rolle's theorem and verify it for the function $f(x) = \frac{x^3}{3} - 3x + 2$ in the interval $[-3, 0]$.

27. State and prove L'Hospital's Rule (First form).

28. State Mean Value Theorem and verify for the function $y = 2x^3 - 3x^2$ in $[1, 2]$.

29. Evaluate $\lim_{x \to 0} \left[\frac{1}{x^2} - \cot^2 x \right]$.

30. Express the solution of the following initial value problem as an integral. $y' = \tan x$, $y(1) = 5$.

31. If f is a continuous function on $[a, b]$, show that :

$$(\min f) \cdot (b - a) \leq \int_a^b f(x)\, dx \leq (\max f) \cdot (b - a).$$

32. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

33. Find the volume, by slicing, of the solid which lies between planes perpendicular to the x-axis at $x = 0$ and $x = 4$. The cross sections perpendicular to the axis on the interval $[0, 4]$ are squares whose diagonals run from the parabola $y = -\sqrt{x}$ to the parabola $y = \sqrt{x}$.

(6 × 5 = 30 marks)

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Part D (Essay Questions)

Answer any **two** *questions (34-36).*
Each question carries 10 marks.

34. Trace the curve $(x^2 + y^2)\, x = a\,(x^2 - y^2),\ a > 0$.

35. State and prove Mean Value Theorem.

36. State and prove Fundamental Theorem of Calculus (Part 1).

(2 × 10 = 20 marks)