D 30568 **(Pages : 3)** **Name...**

Reg. No...................................

FIFTH SEMESTER (CBCSS–UG) DEGREE EXAMINATION NOVEMBER 2022

Mathematics

MTS 5B 05—ABSTRACT ALGEBRA

(2020 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Ceiling is 25.

1. Write addition and multiplication tables for $\mathbb{Z}_4$.

2. Check whether the relation on defined by $a \sim b$ if $n \mid (a - b)$, where n is a positive integer is an equivalence relation.

3. Consider the following permutations in S_7 :

   $$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 5 & 4 & 6 & 1 & 7 \end{pmatrix} \text{ and } \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 5 & 7 & 4 & 6 & 3 \end{pmatrix}.$$

   Compute $\sigma\tau$ and $\tau\sigma$.

4. Show that cancellation property holds in a group G.

5. Find all cyclic subgroups of the group $\mathbb{Z}_6$.

6. Find the order of the element $\begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix}$ in $GL_2(R)$.

7. Give addition table for $\mathbb{Z}_2 \times \mathbb{Z}_2$.

8. Show that composite of two group isomorphisms is a group isomorphism.

9. Give the subgroup diagrams of $\mathbb{Z}_{24}$.

Turn over

10. Find the order of the permutation (1, 2, 5) (2, 3, 4) (5, 6).

11. Let $G = \mathbb{Z}_{12}$, and let H be the subgroup $4\mathbb{Z}_{12}$. Find all cosets of H.

12. Define normal subgroup of a group G. Give an example.

13. Compute the factor group $\dfrac{\mathbb{Z}_6 \times \mathbb{Z}_4}{\langle (2, 2) \rangle}$.

14. Define commutative ring. Give an example.

15. Define Integral Domain. Give an example.

Section B

Answer any number of questions.
Each question carries 5 marks. Ceiling is 35.

16. If $(a, n) = 1$, then show that $a^{\phi(n)} \equiv 1 \pmod{n}$.

17. Let G be a group and let H be a subset of G. Then show that H is a subgroup of G if and only if H is non-empty and $ab^{-1} \in H$ for all $a, b \in H$.

18. Let G be a finite cyclic group with n elements. Show that $G \cong Z_n$.

19. Let $\phi : G_1 \to G_2$ be a group homomorphism with $\text{Ker}\ \phi = \{x \in G_1 : \phi(x) = e\}$. Show that ϕ is one to one if and only if $\text{Ker}\ \phi = \{e\}$.

20. Let G be a group, and let $a, b \in G$ be elements such that $ab = ba$. If the orders of a and b are relatively prime, then prove that $0(ab) = 0(a)\, 0(b)$.

21. Show that any subring of a field is an integral domain.

22. Let G be an abelian group, and let n be any positive integer. Show that the function $\phi : G_1 \to G_2$ defined by $\phi(x) = x^n$ is a homomorphism.

23. State and prove Fundamental Homomorphism Theorem.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*
Maximum 20 *marks.*

24. Show that the inverse of a group isomorphism is a group isomorphism.

25. Show that every sub-group of a cyclic group is cyclic.

26. Let H be a sub-group of the finite group G. Show that the order of H is a divisor of order of G.

27. State and prove First Isomorphism Theorem.