D 11860 **(Pages : 4)** **Name..................................**

Reg. No...............................

THIRD SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MAT 3C 03—MATHEMATICS

(2014—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Define ordinary differential equation.
2. The solution of the differential equation $y' - 1 + y^2$ is ————.
3. The degree of the differential equation $\frac{dy}{dx} = -2 \sin x \cos x$ is ————.
4. The rank of the matrix $A = \begin{bmatrix} 0 & 2 \\ 0 & 5 \end{bmatrix}$ is ————.
5. State True *or* False : The following two matrices are equivalent :

$$\begin{bmatrix} 5 & 5 & -5 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 3 & -3 & 3 & -7 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 3 & -3 & 3 & -7 \end{bmatrix}.$$

6. The characteristic matrix of the matrix $\begin{bmatrix} 0 & 1 \\ 4 & 0 \end{bmatrix}$ is ————.
7. Find the resultant of the vectors **p** = [4, –2, –3], **q** = [8, 8, 1], and **u** = [– 12, – 6, 2].
8. For the vectors **a** = [1, 3, 2], **b** = [2, 0, – 5], and **c** = [4, – 2, 1] , find (**a** · **b**) **c**.

Turn over

9. The vector function $\mathbf{r}(t) = (4 + t)\,\mathbf{i} + (2 + t)\,\mathbf{j}$ represents ————.

10. Let $\vec{r}(t) = 5t^2\,\vec{k}$ be the position vector of a moving particle, where $t \geq 0$ is time. Then the acceleration vector of the moving particle is ————.

11. If $\vec{v} = e^x(\cos y\mathbf{i} + \sin y\mathbf{j})$ then div $\vec{v}$ = ————.

12. When we say that a vector valued function is conservative ?

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Verify that $y = e^{4x}$ is a solution of the differential equation $\frac{dy}{dx} = 4y$.

14. Solve the initial value problem $2xy' - 3y = 0$; $y(1) = 4$.

15. Show that the equation :

$$ydx + xdy = 0$$

is exact and solve it.

16. Reduce the matrix $\begin{bmatrix} 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ to its normal form.

17. Solve completely the system of equations :

$$x + 3y - 2z = 0$$

$$2x - y + 4z = 0$$

$$x - 11y + 14z = 0.$$

18. Find the eigen values of :

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}.$$

19. Prove that $(\mathbf{u} + \mathbf{v})' = \mathbf{u}' + \mathbf{v}'$.

20. Find $\frac{df}{ds}$ in the direction of the vector $4i + 4j - 2k$ at the point (1, 1, 2) if $f(x, y, z) = x^2 + y^2 - z$.

21. Show that $\mathbf{F}(x, y) = (\cos y + y \cos x)\,\mathbf{i} + (\sin x - x \sin y)\,\mathbf{j}$ is a conservative vector field.

22. Find the unit tangent vector at a point t to the curve $\mathbf{r} = a \cos t\,\mathbf{i} + a \sin t\,\mathbf{j}$.

23. Find unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$.

24. Verify that $w = x^2 - y^2$ satisfies Laplace's equation $\nabla^2 w = 0$.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Write in the linear form and then solve $\sin 2x \dfrac{dy}{dx} = y + \tan x$.

26. Determine the rank of the following matrix, by reducing to echelon form :

$$\begin{bmatrix} 1 & a & b & 0 \\ 0 & c & d & 1 \\ 1 & a & b & 0 \\ 0 & c & d & 1 \end{bmatrix}.$$

27. Show that the system of equations :

$$\begin{aligned} x + 2y + z &= 2 \\ 3x + y - 2z &= 1 \\ 4x - 3y - z &= 3 \\ 2x + 4y + 2z &= 4 \end{aligned}$$

is consistent and hence solve them.

28. Find the eigen values and the eigen vector corresponding to the largest eigen value of the matrix

$$\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}.$$

29. Using Cayley-Hamilton theorem find the inverse of $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$.

30. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ with

$$\mathbf{F}(r) = 5z\mathbf{i} + xy\mathbf{j} + x^2z\mathbf{k}$$

along the straight - line segment C : $t\mathbf{i} + t\mathbf{j} + t\mathbf{k}$, $0 \le t \le 1$.

31. Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$ by the divergence theorem where $\mathbf{F} = [x^2, 0, z^2]$ and S is the surface of the box given by the inequalities $|x| \le 1, |y| \le 3, |z| \le 2$.

32. Evaluate the flux integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$ where $\mathbf{F} = \left[e^{2y}, e^{-2z}, e^{2x}\right]$, and

$S : \mathbf{r} = [3 \cos u, 3 \sin u, v], 0 \le u \le \frac{1}{2}\pi, 0 \le v \le 2.$

33. Apply Green's theorem to evaluate $\oint_C (2x^2 - y^2)\, dx + (x^2 + y^2)\, dy$, where C is the boundary of the area enclosed by the x-axis and the upper half off the circle $x^2 + y^2 = a^2$.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Find the orthogonal trajectories of the family of curves $x^2 + y^2 = c^2$.
35. Investigate for what values of a, b the system of equations :

$$x + y + 2z = 2$$
$$2x - y + 3z = 10$$
$$5x - y + az = b$$

have unique solution.

36. Calculate the line integral $\oint_C \mathbf{F}.\mathbf{r}'(s)\, ds$ using Stoke's theorem, where $\mathbf{F} = [-5y, 4x, z]$, and C is the circle $x^2 + y^2 = 4, z = 1$.

(2 × 10 = 20 marks)