C 23878 **(Pages : 4)** **Name..**

Reg. No....................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2022

Mathematics

MTS 2B 02—CALCULUS OF SINGLE VARIABLE—1

(2019—2020 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 25 marks.

1. By reflecting the graph of $y=\sqrt{x}$ sketch the graphs of $y=\sqrt{-x}$ and $y=-\sqrt{x}$.

2. Let $f(x)=x-(\pi/2)$, $g(x)=1+\cos^2 x$, and $h(x)=\sqrt{x}$. Find $h\circ g\circ f$.

3. Find $\lim\limits_{x\to 0}\dfrac{\tan x}{x}$.

4. Find the intervals where the function $f(x)=x^2\sin\left(\dfrac{1}{x}\right)$ is continuous.

5. Find the critical numbers of $f(x)=x-3x^{\frac{1}{3}}$.

6. Verify Rolle's theorem for $f(x)=x^3-x$ in $[-1,1]$.

7. Find the relative extrema of $f(x)=15x^{\frac{2}{3}}-3x^{\frac{5}{3}}$.

8. Define inflection point and state the second derivative test.

Turn over

9. Find the definite integral $\int_0^b x\,dx$ considering it as an area under the graph of a nonnegative function.

10. Find the value of c guaranteed by the Mean Value Theorem for Integrals for $f(x) = 4 - x^2$ on the interval $[0, 3]$.

11. Evaluate $\frac{d}{dx}\int_0^{\sqrt{x}} \cos t\,dt$.

12. Evaluate $\int\limits_{-1}^{1} 5x^4 \sqrt{x^5 + 1}\,dx$.

13. Find the volume of the solid obtained by revolving the region under the graph of $y = \sqrt{x}$ on $[0, 2]$ about the x-axis.

14. Find the work done by the force $F = 3x^2 + x$ (measured in pounds) in moving a particle along the x-axis from $x = 2$ to $x = 4$ (measured in feet).

15. Define the Center of Mass of a System of Masses on a Line and The Center of Mass of a System of Particles in a Plane.

Section B

Answer any number of questions.
Each question carries 5 *marks.*
Maximum 35 *marks.*

16. Prove that if f is differentiable at a, then f is continuous at a.

17. Suppose the weekly revenue realized through the sale of x Pulsar cell phones is

$$R(x) = -0.000078x^3 - 0.0016x^2 + 80x \qquad 0 \le x \le 800 \qquad \text{dollars.}$$

(i) Find the marginal revenue function.

(ii) If the company currently sells 200 phones per week, by how much will the revenue increase if sales increase by one phone per week ?

18. During a test dive of a prototype of a twin-piloted submarine, the depth in feet of the submarine at time t in minutes is given by $h(t) = t^3 (t-7)^4$ where $0 \le t \le 7$. Find the inflection points of h.

19. Prove that $\lim_{x \to 0} \frac{1}{x^2} = \infty$.

20. State The Fundamental Theorem of Calculus, Part 2 and evaluate

$$\int_{-2}^{2} f(x)\,dx, \text{ where } f(x) = \begin{cases} -x^2 + 1 & \text{if } x < 0 \\ x^3 + 1 & \text{if } x \ge 0 \end{cases}.$$

21. A car moves along a straight road with velocity function $v(t) = t^2 + t - 6 \quad 0 \le t \le 10$ where $v(t)$ is measured in feet per second.

    (i) Find the displacement of the car between $t = 1$ and $t = 4$.

    (ii) Find the distance covered by the car during this period of time.

22. Find the length of the graph of $x = \frac{1}{3}y^3 + \frac{1}{4y}$ from $P\left(\frac{7}{12}, 1\right)$ to $Q\left(\frac{67}{24}, 2\right)$.

23. Find the area of the surface obtained by revolving thegraph of $y = \sqrt{x}$ on the interval $[0, 2]$ about the x-axis.

Section C

Answer any **two** *questions.*
Each question carries 10 *marks.*
Maximum 20 *marks.*

24. (a) Show that the function f defined by $f(x) = \sqrt{4 - x^2}$ is continuous on the closed interval $[-2, 2]$.

    (b) Assume that the moon is a perfect sphere, and suppose that we have measured its radius and found it to be 1080 mi with a possible error of 0.05 mi. Estimate the maximum error in the computed surface area of the moon.

25. (a) Let $f(x) = \dfrac{2x^2 - x + 1}{3x^2 + 2x - 1}$. Find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$, and find all horizontal asymptotes of the graph of *f*.

(b) A man has 100 ft of fencing to enclose a rectangular garden in his backyard. Find the dimensions of the garden of largest area he can have if he uses all of the fencing.

26. (a) By computing Riemann sum, evaluate $\int_{-1}^{3} \left(4 - x^2\right) dx$.

(b) Find the average value of $4 - x^2$ over the interval $[-1, 3]$.

27. (a) Prove that the length *s* of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ measured from $(0, a)$ to the point (x, y) is given by $s = \dfrac{3}{2} \sqrt[3]{ax^2}$. Also find the entire length

(b) Find the area of the surface generated by revolving the arc of the catenary $y = c \cosh \dfrac{x}{c}$ from $x = 0$ to $x = c$ about the x-axis.

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SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2022

Mathematics

MTS 2B 02—CALCULUS OF SINGLE VARIABLE—1

(2019—2020 Admissions)

(Multiple Choice Questions for SDE Candidates)

Time : 15 Minutes	**Total No. of Questions : 20**	**Maximum : 20 Marks**

INSTRUCTIONS TO THE CANDIDATE

1. This Question Paper carries Multiple Choice Questions from 1 to 20.
2. The candidate should check that the question paper supplied to him/her contains all the 20 questions in serial order.
3. Each question is provided with choices (A), (B), (C) and (D) having one correct answer. Choose the correct answer and enter it in the main answer-book.
4. The MCQ question paper will be supplied after the completion of the descriptive examination.

MTS 2B 02—CALCULUS OF SINGLE VARIABLE—1

(Multiple Choice Questions for SDE Candidates)

1. Which among the following is an even function ?

   (A) $\sin(x)$.
   (B) $\tan(x)$.
   (C) $x^4 - x^2 + 1$.
   (D) x^5.

2. $\lim_{x \to 0} \frac{\tan(x)}{x}$ is :

   (A) 0.
   (B) 1.
   (C) π.
   (D) Limit does not exists.

3. The following statement is true : "If $f(x) = \sin(x)$, then f is a continuous function." Which of the following is also true ?

   (A) If $f(x)$ is not equal to $\sin x$, then f is not continuous.
   (B) If f is not a continuous function, then $f(x)$ is not equal to $\sin x$.
   (C) If f is continuous, then $f(x) = \sin(x)$.
   (D) None of these.

4. $\lim_{x \to a} f(x) = \mathrm{L}$ if and only if :

   (A) $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = \mathrm{L}$.
   (B) $f(a) = \mathrm{L}$.
   (C) $\lim_{x \to a^-} f(x) = \mathrm{L}$.
   (D) None of the above.

5. On $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right], f(x) = \cos(x)$ takes on :

(A) A maximum value of 1 (once) and a minimum value of 0 (twice).

(B) A maximum value of 1 (once) and no minimum value.

(C) A minimum value of 0 (twice) and no maximum value.

(D) A maximum value of 1 (once) and a minimum value of 0 (once).

6. Consider the function

$$f(x) = \begin{cases} x+1 & -1 \le x < 0 \\ 0 & x = 0 \\ x-1 & 0 < x \le 1 \end{cases}$$

Then which of the following statements is NOT true ?

(A) f is continuous at every point of $[-1, 1]$, except at $x = 0$.

(B) f has a non-removable discontinuity at $x = 0$.

(C) f has neither a highest nor a lowest point on $[-1, 1]$.

(D) f has the highest value 1 and the lowest value -1 on $[-1, 1]$.

7. The only domain points where a function can assume extreme values are ——————.

(A) Critical points and end points. (B) Critical points only.

(C) End points only. (D) None of the above.

8. Using which of the following reasons, can we conclude that "The Rolle's theorem cannot be applied to the function $f(x) = \tan x$ for the interval $[0, \pi]$."

(i) There is a discontinuity at $x = \frac{\pi}{2}$ to the function $f(x) = \tan x$.

(ii) $f'(x) = \sec^2 x$ which does not exist at $x = \frac{\pi}{2}$.

(A) Both (i) and (ii). (B) (i) only.

(C) (ii) only. (D) None of the above.

9. At a critical point c, if f' changes from positive to negative at $c\left(f' > 0 \text{ for } x < c \text{ and } f' < 0 \text{ for } x > c\right)$, then f has ——————.

(A) A local maximum value at c.
(B) A local minimum value at c.
(C) Global minimum value at c.
(D) None of the above.

10. A curve is said to be concave upwards (or convex downwards) at or near P when at all points near P on it ——————.

(A) Lies *above* the tangent at P.
(B) Lies *below* the tangent at P.
(C) Lies *on* the tangent at P.
(D) None of these.

11. $\lim_{x \to \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2} =$ ——————.

(A) $\frac{5}{3}$.
(B) 5.
(C) 0.
(D) ∞.

12. Use the linear approximation of $f(x) = \sqrt{1+x}$ at $a = 0$ to estimate $\sqrt{0.95}$.

(A) 0.942.
(B) 0.995.
(C) 0.9820.
(D) 0.9750.

13. The linearization of a function $f(x) = \cos(2x)$ at the $x = \frac{1}{2}$.

(A) $y = \cos(1) - 2\sin(1)\left(x - \frac{1}{2}\right)$.
(B) $y = \cos(2) - 2\sin(2)\left(x - \frac{1}{2}\right)$.
(C) $y = \frac{1}{2} - 2\sin(1)\left(x - \frac{1}{2}\right)$.
(D) $y = \cos(1) + \sin(1)\left(x - \frac{1}{2}\right)$.

14. If $y = 9x^2 - 4x + 3$, then $\frac{d^2y}{dx^2}$ is :

(A) $18x - 4$.
(B) -4.
(C) 22.
(D) 18.

15. If $y = x^5 + 37x$, then dy is ————————.

(A) $dy = \left(5x^4 + 37x\right) dx.$

(B) $dy = \left(5x^4 + 37\right) dx.$

(C) $dy = \left(\frac{x^6}{6} + 37\frac{x^2}{2}\right) dx.$

(D) $dy = \left(3x^4 + 37\right) dx.$

16. The radius r of a circle increases from $r_0 = 10$ m to 10.1 m. Estimate the increase in the circle's area A by calculating dA :

(A) $dA = 2\pi \text{ m}^2.$

(B) $dA = -2\pi \text{ m}^2.$

(C) $dA = \pi \text{ m}^2.$

(D) $dA = -\pi \text{ m}^2.$

17. Define norm of a partition :

(A) The norm of a partition P is the first subinterval length.

(B) The norm of a partition P is the average of partition's subinterval length.

(C) The norm of a partition P is the partition's shortest subinterval length.

(D) The norm of a partition P is the partition's longest subinterval length.

18. If $f(x) \geq g(x)$ on $[a, b]$, then :

(A) $\int_a^b f(x)\, dx \geq \int_a^b g(x)\, dx.$

(B) $\int_a^b f(x)\, dx \leq \int_a^b g(x)\, dx.$

(C) $\int_a^b f(x)\, dx = \int_a^b g(x)\, dx.$

(D) $\int_a^b f(x)\, dx = -\int_a^b g(x)\, dx.$

19. If f is integrable on $[a, b]$, its average(mean) value on $[a, b]$ is ————————.

(A) $av(f) = \int_a^b f(x)\, dx.$

(B) $av(f) = \frac{1}{b-a}\int_a^b f(x)\, dx.$

(C) $av(f) = \frac{1}{a-b}\int_a^b f(x)\, dx.$

(D) $av(f) = \frac{f(x)}{b-a}.$

20. $\int_0^\pi \cos x\, dx =$ ————————.

(A) 1.

(B) $\frac{1}{2}$.

(C) 0.

(D) – 1.