D 100615 **(Pages : 3)** **Name...................................**

Reg. No.................................

SIXTH SEMESTER UG (CBCSS-UG) DEGREE EXAMINATION, MARCH 2024

Mathematics

MTS 6B 13—DIFFERENTIAL EQUATIONS

(2019 Admission onwards)

Time : Two Hours and a Half Maximum Marks : 80

Section A (Short Answer Type Questions)

Answer any number of questions.
Each carry 2 marks. Maximum marks 25.

1. State Existence and Uniqueness Theorem for First Order Linear Differential Equations.
2. Determine the values of r for which e^{rt} is a solution of the differential equation $y''' - 3y'' + 2y' = 0$.
3. Using method of integrating factors solve the differential equation $\frac{dy}{dt} - 2y = 4 - t$.
4. Show that the given differential equation is exact :

$$\left(x^3 + 3xy^2\right)dx + \left(3x^2 y + y^3\right)dy = 0.$$

5. Find the Wronskian of the functions $e^{\lambda_1 x}, e^{\lambda_2 x}$.
6. Solve the differential equation $y'' - 2y' - 3y = 3e^{2t}$.
7. Let $y = \phi(x)$ be a solution of the initial value problem

$$\left(1 + x^2\right)y'' + 2xy' + 4x^2 y = 0,\ y(0) = 0,\ y'(0) = 1.$$

Determine $\phi'''(0)$.

8. Determine a lower bound for the radius of convergence of series solutions about each given point $x_0 = 4$ for the given differential equation $y'' + 4y' + 6xy = 0$.
9. Find the Laplace transform of $2t + 6$.
10. Find the inverse Laplace transform of $\frac{s-4}{s^2+4}$.

Turn over

11. If $F(s) = \mathcal{L}(f(t))$ exists for $s > a \geq 0,$ and if c is a constant. Show that

$$\mathcal{L}\left(e^{ct} f(t)\right) = F(s-c), s > a + c.$$

12. If $\mathcal{L}(f)$ denote the Laplace transform of the function $f(x)$. Show that

$$\mathcal{L}(f_1 + f_2) = \mathcal{L}(f_1) + \mathcal{L}(f_2), \; \mathcal{L}(cf) = c\mathcal{L}(f).$$

13. Solve the boundary value problem :

$$y'' + y = 0, y(0) = 1, y(\pi) = a.$$

14. Define an even function and show that if $f(x)$ is an even function then

$$\int_{-L}^{L} f(x)\, dx = 2\int_{0}^{L} f(x)\, dx.$$

15. Verify that the method of separation of variables may be used to solve the equation $xu_{xx} + u_t = 0.$

Section B (Paragraph/Problem)

Answer any number of questions.
Each carry 5 *marks. Maximum marks* 35.

16. Show that the equation $\frac{dy}{dx} = \frac{x^2}{1-y^2}$ is separable, and then find an equation for its integral curves.

17. Find the value of b for which the following equation is exact, and then solve it using that value of b.

$$\left(xy^2 + bx^2 y\right) + (x+y)x^2 y' = 0.$$

18. Solve the initial value problem $y'' + 4y = t^2 + 3e^t, y(0) = 0, y'(0) = 2.$

19. Find the general solution of the differential equation $y'' + y = \tan t$ on $0 < t < \pi/2.$

20. Using Laplace transform solve the initial value problem :

$$y'' + 4y = 0, y(0) = 3, y'(0) = -1.$$

21. Find the inverse Laplace transform of the following function using the convolution theorem :

$$F(s) = \frac{1}{(s+1)^2 \left(s^2 + 4\right)}.$$

22. Determine the coefficients in the Fourier series of the function

$$f(x)=\begin{cases}-x, & -2\le x\le 0,\\ x, & 0\le x<2\end{cases}$$

with $f(x+4)=f(x)$.

23. Find the solution of the following heat conduction problem :

$$\begin{aligned} &100u_{xx}=u_t, && 0<x<1,\ t>0;\\ &u(0,t)=0,\ u(1,t)=0, && t>0;\\ &u(x,0)=\sin(2\pi x)-\sin(5\pi x), && 0\le x\le 1.\end{aligned}$$

Section C (Essay Type Questions)

Answer any **two** *questions.*
Each carry 10 *marks.*

24. Find the general solution of the following differential equaton using the method of integrating factors

$$\frac{dy}{dt}+\frac{1}{2}y=\frac{1}{2}e^{t/3}.$$

Draw some representative integral curves of the differential equation and also find the particular solution whose graph contains the point (0, 1).

25. Find a series solution of the differential equation :

$$y''+y=0,\ -\infty<x<\infty.$$

26. Find the Laplace transform of $\int_0^t \sin(t-\tau)\cos\tau\, d\tau$.

27. Find the Fourier series of the following periodic function $f(x)$ of period p = 2L defined by

$$f(x)=3x^2\ -1<x<1.$$

(2 × 10 = 20 marks)