C 41231 **(Pages : 4)** **Name.......................................**

Reg. No..................................

FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2023

Mathematics

MTS 4B 04—LINEAR ALGEBRA

(2019 Admission onwards)

Time : Two Hours and a Half — Maximum : 80 Marks

Section A (Short Answer type Question)

Each question carries 2 *marks.*
All questions can be attended.
Overall ceiling 25.

1. Give an example of a system of linear equation with the following properties :

   (i) Unique solution ; and

   (ii) No solution.

2. For any 2 × 2 matrices, A and B, prove that

   $\text{trace}(A+B)=\text{trace}(A)+\text{trace}(B)$.

3. Define all subspaces of the vector space $\mathbb{R}^3$ over $\mathbb{R}$.

4. Define linear combination of vectors in a vector space. Write $(2,3)$ as the linear combination of $(1,0)$ and $(0,1)$.

5. Define basis of a vector space. Write a basis of P_n, where P_n is the polynomials of degree less than or equal to n.

6. Consider the basis $B=\{u_1, u_2\}$ and $B'=\{u_1', u_2'\}$ of $\mathbb{R}^2$, where $u_1=(1,0)$, $u_2=(0,1)$, $u_1'=(1,1)$ and $u_2'=(2,1)$. Find the transformation matrix from $B' \to B$.

7. Let $W=\{(x, y)\in\mathbb{R}^2 : x+y=0\}$. Find the dimension of W.

Turn over

8. Give an example of an infinite dimensional vector space.

9. Define rank and nullity of a matrix.

10. Find the image of $x = (1, 1)$ under the rotation of $\frac{\pi}{6}$, about the origin.

11. Define eigen values and eigen vectors of a matrix.

12. Find the egien values of $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & 2/3 & 0 \\ 5 & -8 & -\frac{1}{4} \end{bmatrix}$.

13. If λ is the eigen values of a matrix A, show that λ^n is the eigen values of A^{n}.

14. Show that $(1, 1)$ and $(1, -1)$ are orthogonal vectors with respective the Euclidian inner product.

15. Let W be the subspace spanned by the orhonornal vector $v_1 = (0, 1, 0)$. Find the orthogonal projection of $u = (1, 1, 1)$ on W.

(Ceiling 25 marks)

Section B (Paragraph/Problem Type Questions)

Each question carries 5 marks.
All questions can be attended.
Overall Ceiling 35.

16. Solve the following linear system by Gauss-Elimination method,

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 8 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - 7x_2 + 4x_3 &= 10. \end{aligned}$$

17. Prove that, if A and B are invertible matrices of same size, then AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.

18. Show that the set $\{(1,1,2),(1,0,1),(2,1,3)\}$ spans $\mathbb{R}^3$.

19. Show that the operator $T:\mathbb{R}^2 \to \mathbb{R}^2$ defined by the equations

$$w_1 = 2x_1 + x_2$$
$$w_2 = 3x_1 + 4x_2$$

is one-one, and find $T^{-1}(w_1, w_2)$.

20. Let T be the operator which is the reflection about the *xz* plane in $\mathbb{R}^3$. Find the matrix of T with respective the standard basis.

21. Find the rank and nullity of the matrix

$$\begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}.$$

22. Find the bases of the eigen spaces of the matrix

$$\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}.$$

23. Show that a square matrix A is invertible if and only if 0 is not and eigen value of A.

(Ceiling 35 marks)

Section C (Essay Type Question)

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 \\ 2 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

(b) Define the followings with examples :

(i) Diagonal matrices ;

(ii) Lower triangular matrices ;

(iii) Upper triangular matrices ;

(iv) Symmetric matrices ; and

(v) Singular matrices.

25. Let $v_1 = \{1, 2, 1\}, v_2 = \{2, 9, 0\}$ and $v_3 = \{3, 3, 4\}$.

(a) Show that $\{v_1, v_2, v_3\}$ is a basis of $\mathbb{R}^3$.

(b) Find the co-ordinate vector of $v = (5, -1, 9)$ relative to the basis $\{v_1, v_2, v_3\}$.

26. Consider the following linear system :

$$\begin{bmatrix} -1 & 3 & 2 \\ 1 & 2 & -3 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ -3 \end{bmatrix}.$$

(a) Show that the above system is consistent.

(b) Solve the above system of linear equations.

27. (a) Define similar matrices.

(b) Show that the following matrix is not diagonazible :

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{bmatrix}.$$

(2 × 10 = 20 marks)